An Introduction to the Theory of Predicting Noise from Turbulence with a Historical Perspective

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Outline

- Introductory material
- Noise from jets and need for reduction
- Early developments of Lighthill
- Improvements of Lilley
- The industrious model of Ffowcs-Williams
- An alternative approach of Tam
- Origins of jet source separation
- A new model
- Concluding remarks

Biography - Miller

Experience

- MAE UF since August 2016
- Seven years NASA Langley Research Aerospace Engineer
- Ph.D. / M.S. Aerospace Engineering at Penn State
- Tagonrog State University, Russia (statistics)
- B.S. Mechanical Engineering at Michigan State

Personal

- Origination in Michigan
- Art, pure math, scientific history

Figure 1: Miller at University of Florida Library Reading Room

Theoretical Fluid Dynamics and Turbulence Group

- Fluid Dynamics and Turbulence - focus on theory and numerics
- Jet noise
- Fluid structure interaction
- Sonic boom / shock wave propagation
- Rotorcraft and rotor-aeroacoustics
- New research - tornadoes
- Where curiosity leads us

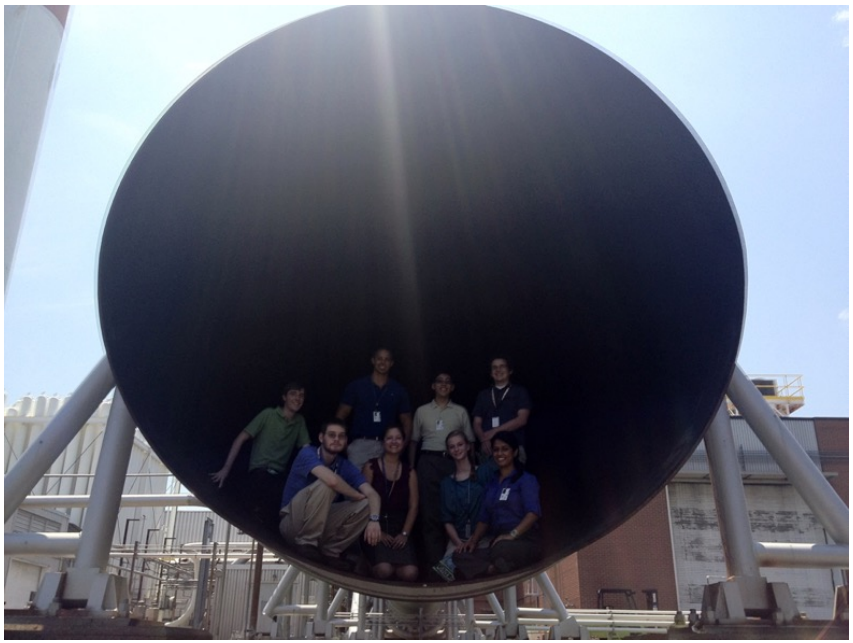


Figure 2: Students at NASA Langley 8 Ft. Hypersonic Tunnel

Early Aeroacoustics Phenomena

- Studying stability of jet flows dates from the nineteenth century
- Playing ‘grand trios of Beethoven’ in 1858, Leconte noticed that cello altered coal-gas jets used for lighting
- Tyndall conducted experiments and noted smooth flame of 24 in. reduces to 7 in. due to keys shaken
- Transient irregularities on surface of smooth jet flame is created by noise

Figure 3: Flame from Sandia (http://crf.sandia.gov).

World War II and the Jet Engine

- Germany creates and deploys first production jet aircraft, the Messerschmitt Me 262
- WWII Germany conducted first acoustic measurements of jet engines
- Microphones mounted within wings
- Noise contributed to detection (from German point of view)
- Noise from jet flows became a societal problem

Figure 4: Messerschmitt Me 262, the world's first jet fighter (photo U.S. Air Force).

Contemporary Jet Noise Problems

- US Navy
- Commercial aviation
- US Gov. - Hard to determine cost estimates - over 1 billion dollars per year
- Tinnitus and hearing loss most prevalent disabilities in service of country (157,000+85,000) people respectively

Figure 5: JSF take-off courtesy of U.S. ONR (see Martens [1])

Figure 6: QDT2 Test (NASA.gov).

Contemporary Rocket Noise and Associated Problems

- Noise from rockets considerable problem
- Noise source is rocket exhaust
- Vibrations and failure of launch vehicle
- Damage to launchpad structure
- Often reduced through water injection
- Scale model and full-scale measurements of noise from rockets

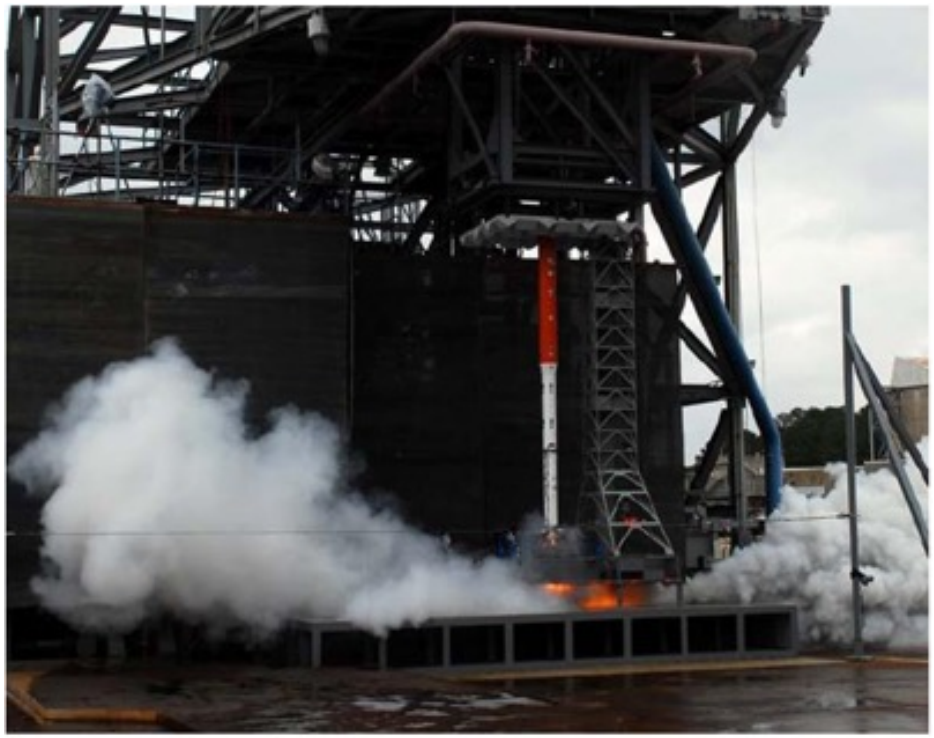


Figure 7: Scale model test at Marshall Space Flight Center of Counter and Houston [2].

Physics of Turbulent Supersonic Jets

The jet flow-field is very complicated containing subsonic, transonic, and supersonic flows, shocks, expansions, turbulence, heating, and sometimes chemistry.

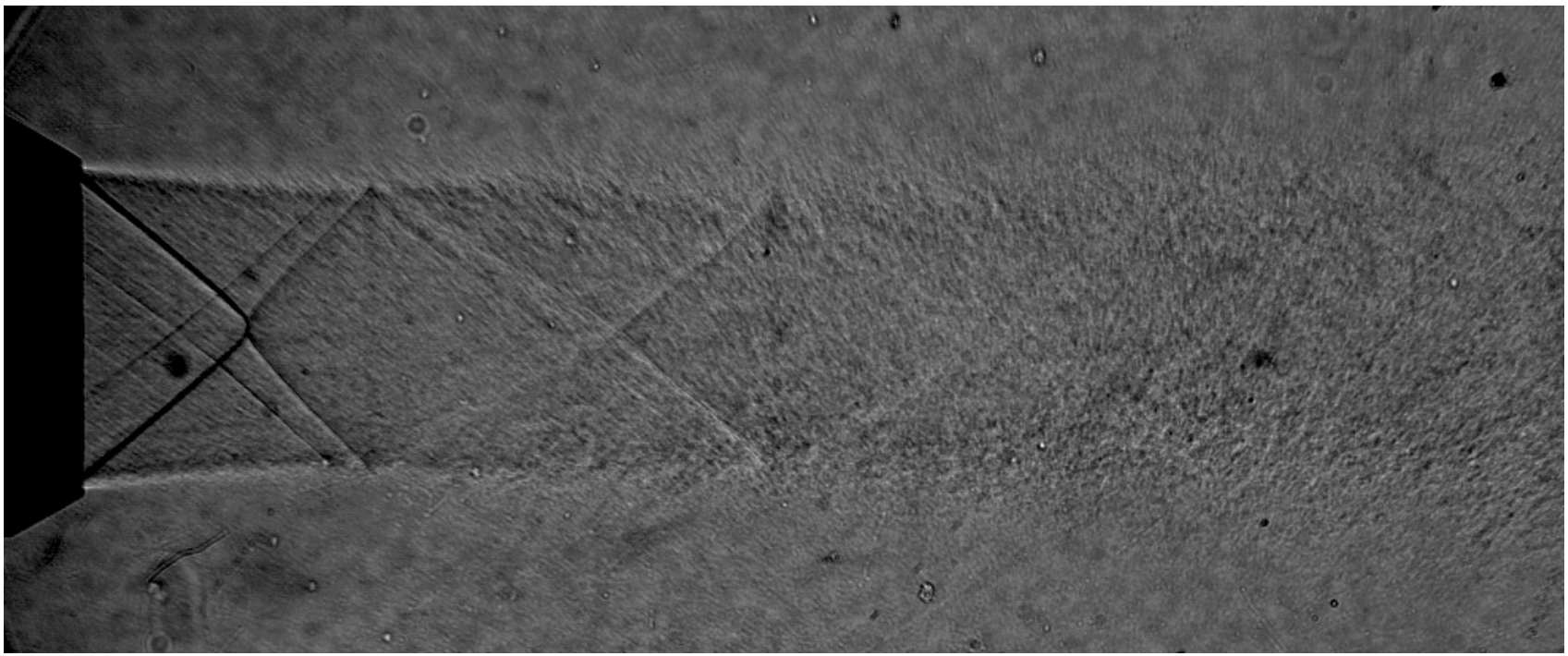


Figure 8: Schlieren of off-design supersonic jet courtesy of Zaman et al. [3].

Jet Noise Acoustic Spectra and Sound Sources

- Acoustic spectra of turbulent jets are very complicated
- Flow-field phenomena correspond to different noise sources
- Each source radiates in unique directions and has unique characteristics
- Noise sources are mixing, shock, Mach wave radiation, tones, and more

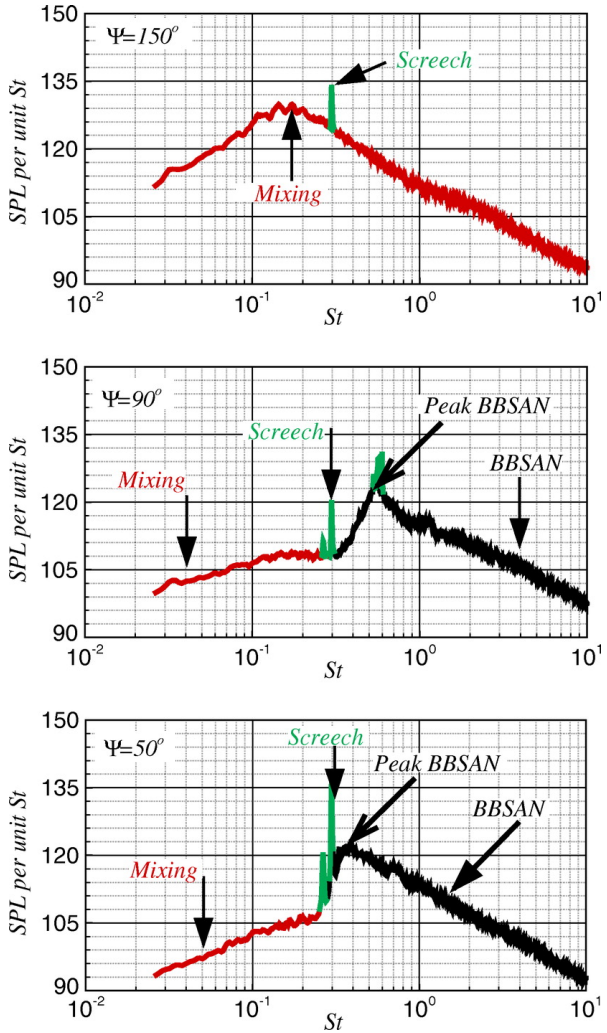


Figure 9: Jet noise spectral decomposition by source of Miller [4].

George Green
July 14, 1793 – May 31, 1841, British

- British Miller and Mathematician
- Green's theorem, potential functions, and Green's functions
- Created a mathematical theory of electricity and magnetism, formed the foundation for the work of other scientists
- Entirely self-taught with approx. 1 year of school when 8 years old
- Had common law wife Jane Smith with 7 children

AN ESSAY

MATHEMATICAL ANALYSIS TO THE THEORIES OF ELECTRICITY AND MAGNETISM.

BY

GEORGE GREEN.

S.A.E. Miller, Ph.D., saem@ufl.edu

Green's Function I

- To solve partial differential equations we can use the theory of vector Green's functions
- We illustrate the idea with a simple linear ODE

Consider the equation

$$T\frac{\partial^2 u}{\partial x^2} = f(x) \tag{1}$$

where T is a constant and $f(x)$ is a source function on x. Seek a solution for u by defining the Green's function from a source $\delta(x-\xi)$

$$T\frac{\partial^2 g}{\partial x^2} = \delta(x-\xi) \tag{2}$$

Recall the definition of the Dirac delta (δ) generalized function

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \tag{3}$$

Green's Function II

with

$$\delta(t) = \infty \text{ for } t = 0 \text{ and } \delta(t) = 0 \text{ for } t \neq 0 \tag{4}$$

We set boundary conditions for u and find subsequent boundary conditions for g (they are sometimes different).
We solve for g

$$\begin{aligned} g(x|\xi) &= ax + b \text{ for } 0 \leq x < \xi \\ g(x|\xi) &= cx + d \text{ for } \xi < x \leq L \end{aligned} \tag{5}$$

We can now write the solution as

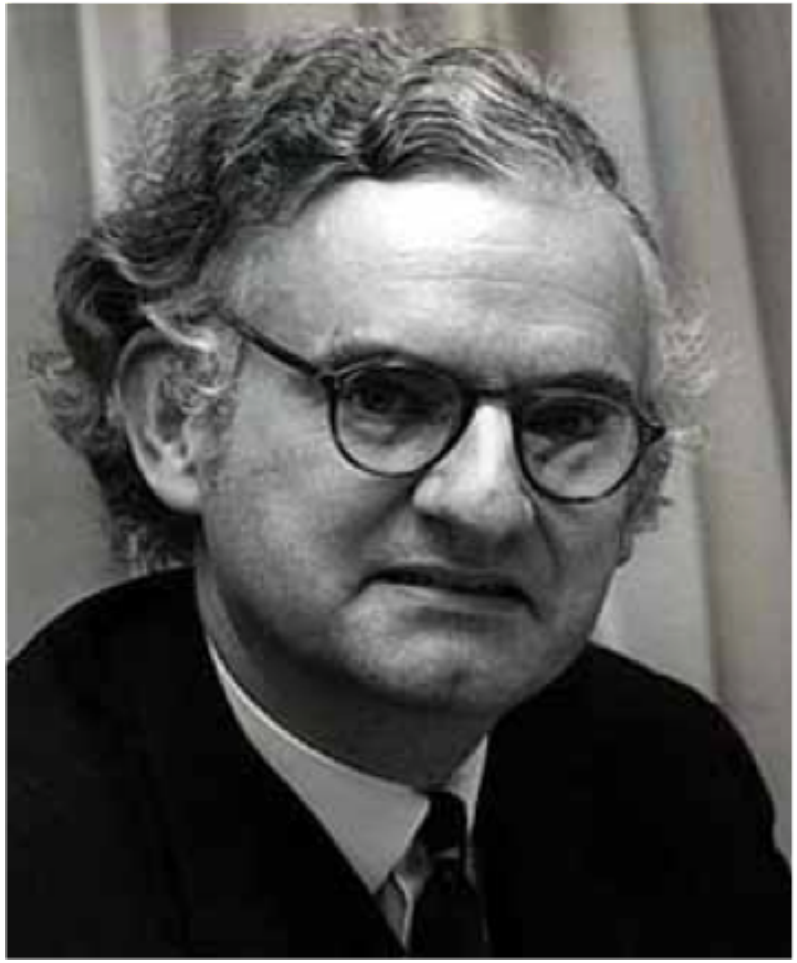
$$u(x) = \int_0^L f(\xi) g(x|\xi) d\xi \tag{6}$$

- For an introduction see Duffy, D. G. "Green's Functions with Applications."

Sir Michael James Lighthill

23 January 1924 – 17 July 1998, British

- Applied Mathematician
- Created acoustic analogy
- Worked on aeroacoustics, fluid dynamics, supersonic flight (Concorde), manned spacecraft
- Lucasian Professor of Mathematics, succeeded by Prof. Stephan Hawking
- Lighthill report led to AI Winter in the UK
- Open water swimmer



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Lighthill Acoustic Analogy I

- Lighthill, in his now famous works on general theory [5] and turbulence as a source of sound [6], introduced the acoustic analogy.

Combining the time derivative of the continuity equation with the divergence of the momentum equation, an inhomogeneous wave equation results from the Navier-Stokes equations,

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 \mathcal{T}_{ij}}{\partial x_i \partial x_j}, \tag{7}$$

where $\mathcal{T}_{ij} = \rho u_i u_j - \tau_{ij} + (p - \rho c_\infty^2)\delta_{ij}$ is the Lighthill stress tensor.

- Lighthill constructed a model for $\mathcal{T}_{ij}$ consisting of convecting quadrupoles that resulted in an estimate of jet noise intensity scaling as u_j^8.
- That is really nice but we need to solve this equation ...

Solution of Lighthill Acoustic Analogy

- Solution for ρ is found by the convolution integral of the Green's function of the wave equation with the double divergence of $\mathcal{T}_{ij}$.

We seek the Green's function of the wave equation

$$\frac{\partial^2 g}{\partial t^2} - c_\infty^2 \frac{\partial^2 g}{\partial x_i \partial x_i} = \delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - \tau), \tag{8}$$

which is $g(x; y, t; \tau) = \frac{\delta(t-\tau-|\boldsymbol{x}-\boldsymbol{y}|/c_\infty)}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}$. The density field is

$$\rho - \rho_\infty = \frac{1}{4\pi c_\infty^2} \frac{\partial^2}{\partial x_i \partial x_j} \int\int\int \mathcal{T}_{ij}\left(\boldsymbol{y}, t - \frac{|\boldsymbol{x}-\boldsymbol{y}|}{c_\infty}\right) \frac{d\boldsymbol{y}}{|\boldsymbol{x}-\boldsymbol{y}|}. \tag{9}$$

Geoffrey Michael Lilley
16 November 1919 – 20 September 2015, British

- Order of the British Empire and aeronautical scientist
- Worked extensively in experimental, mathematical, aeroacoustics and aerodynamics
- Created first ‘chevrons’ or ’teeth’ used in wind-tunnels and now on nozzles
- Worked in field for 7 decades and some consider the father of the field
- Examined sonic boom, supersonic aircraft, jet noise extensively, silent flight of owls
- Married Lesley Wheeler and had three children

National Air and Space Museum
Smithsonian Institution

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Chevrons - Heritage of Professor G. M. Lilley

- Developed chevrons or ‘teeth’ for open jet wind tunnel during WWII
- First to apply chevrons to jet nozzle exit
- Chevrons alter flow-field structure by inducing streamwise vortices
- Careful design lowers noise and hopefully maintains performance
- Implementation on full-scale tactical aircraft on-going and successful on commercial engines

Figure 10: Chevrons on military engine test (see Martens [1])

Lilley's Acoustic Analogy I

Lilley [7] developed a very important acoustic analogy following Lighthill [5, 6]
Lilley's equation is

$$\frac{\overline{D}^3 r'}{Dt^3} - \frac{\overline{D}}{Dt}\left(\overline{c}^2 \bigtriangledown^2 r'\right) - \frac{\partial \overline{c}^2}{\partial y_2}\frac{\overline{D}}{Dt}\frac{\partial r'}{\partial y_2} + 2\overline{c}^2 \frac{\partial \overline{u}_1}{\partial y_2}\frac{\partial^2 r'}{\partial y_1 \partial y_2} = \Lambda\left(\boldsymbol{y}, t\right) \quad (10)$$

where $r' = (1/\gamma)\ln p$ and $\overline{D}/Dt = \partial/\partial t + \overline{u}_1 \partial/\partial x_1$.

- Λ is a nonlinear source term that involves the base flow and perturbation quantities and does not contain r'.
- Major advantage relative to Lighthill - sound refraction and propagation due to turbulence are accounted for and solution is not part of source, but assumed parallel meanflow

John Eirwyn Ffowcs-Williams
25 May 1935 – , UK

- Based on Lighthill acoustic analogy developed the Ffowcs-Williams Hawkings method
- For 25 years he led the division in which The University Cambridge's Fluid Mechanics, Aeronautics, Thermodynamics, and Turbomachinery
- Encouraged students to think creatively without critism

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Ffowcs-Williams Hawkings Acoustic Analogy I

- Ffowcs-Williams and Hawkings (FWH) [8] constructed generalized forms of the continuity and momentum equations involving two regions of the flow-field divided by the Heaviside function
- Solution resulted in three source terms associated with traditional acoustic theory - monopoles, dipoles, and quadrupoles
- Resulted in ability to predict noise from moving surfaces in arbitrary motion (eg helicopter blades, propellers, etc.)

The FWH equation (acoustic analogy) is

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i \partial x_i}\right) p' = \frac{\partial}{\partial t}\left[\rho_\infty \mathcal{U}_{n'}\right]\delta(g) - \frac{\partial}{\partial x_i}\left[p n_i' \delta(g)\right] + \frac{\partial^2}{\partial x_i \partial x_j}\left[\mathcal{T}_{ij}\mathcal{H}(g)\right] \tag{11}$$

Ffowcs-Williams Hawkings Acoustic Analogy II

Written in the more convenient way of Farassat [9]

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i \partial x_i}\right) p' = \frac{\partial}{\partial t}\left[\rho_\infty U_{n'}\right]\delta(g) - \frac{\partial}{\partial x_i}\left[L_i \delta(g)\right] + \frac{\partial^2}{\partial x_i \partial x_j}\left[\mathcal{T}_{ij}\mathcal{H}(g)\right], \tag{12}$$

where U and L are thickness and loading terms.

- Farassat [10] derived his Formulation 2B from Eqn. 11 to predict pressure in the far-field from the FWH surface
- Define a closed-surface around the source (turbulence, propeller, airplane, submarine, butterfly, etc.)
- Now seek a solution through the Green's function and theory of generalized functions

Ffowcs-Williams Hawkings Acoustic Analogy III

We find the solution of Eqn. 12 as

$$4\pi p'(x,t) = \int_{-\infty}^{\infty} \left[\frac{\rho_\infty \dot{\mathcal{U}}_{n'}}{r(1-M_r)^2} + \frac{\rho_\infty \mathcal{U}_{n'}(r\dot{M}_r + c(M_r - M^2))}{r^2(1-M_r)^3} + \frac{\dot{\ell}_r}{cr(1-M_r)^2} + \frac{\ell_r - \ell_i \cdot M_i}{r^2(1-M_r)^2} \right.$$
$$+ \frac{\ell_r(r\dot{M}_r + c(M_r - M^2))}{cr^2(1-M_r)^3} + \left(\frac{\dot{\rho}(\mathcal{V}_{n'} - \mathcal{U}_{n'}) + \rho(\dot{\mathcal{V}}_{n'} - \dot{\mathcal{U}}_{n'})}{r(1-M_r)^2} \right)(1+\mathcal{M}_r) + \frac{\rho(\mathcal{V}_{n'} - \mathcal{U}_{n'})}{r(1-M_r)^2}\dot{\mathcal{M}}_r$$
$$\left. + \frac{\rho(\mathcal{V}_{n'} - \mathcal{U}_{n'})(\mathcal{V}_r - \mathcal{V}_i M_i)}{r^2(1-M_r)^2} + \frac{\rho(\mathcal{V}_{n'} - \mathcal{U}_{n'})(1+\mathcal{M}_r)}{r^2(1-M_r)^3}(r\dot{M}_r + c(M_r - M^2)) \right]_{ret} dS, \tag{13}$$

where $\boldsymbol{M}$ and $\boldsymbol{\mathcal{M}}$ denote the Mach number of the surface velocity and flow velocity, respectively. We can substitute $\boldsymbol{\mathcal{U}} = 0$ and $\boldsymbol{M} = 0$ into Eqn. 13 and simplify

$$4\pi p'(x,t) = \int_{-\infty}^{\infty} \left[\frac{\dot{\ell}_r}{cr} + \frac{\ell_r}{r^2} + \left(\frac{\dot{\rho}\mathcal{V}_{n'} + \rho\dot{\mathcal{V}}_{n'}}{r} \right)(1+\mathcal{M}_r) \right.$$
$$\left. + \frac{\rho\mathcal{V}_{n'}}{r}\dot{\mathcal{M}}_r + \frac{\rho\mathcal{V}_{n'}\mathcal{V}_r}{r^2} \right]_{ret} dS. \tag{14}$$

Example of Ffowcs-Williams and Hawkings Jet Noise Prediction

- Most predominant contemporary approach
- Illustrative example of jet noise prediction using LES from Wang and Miller [11]
- Dedicated research groups can perform a much more accurate prediction

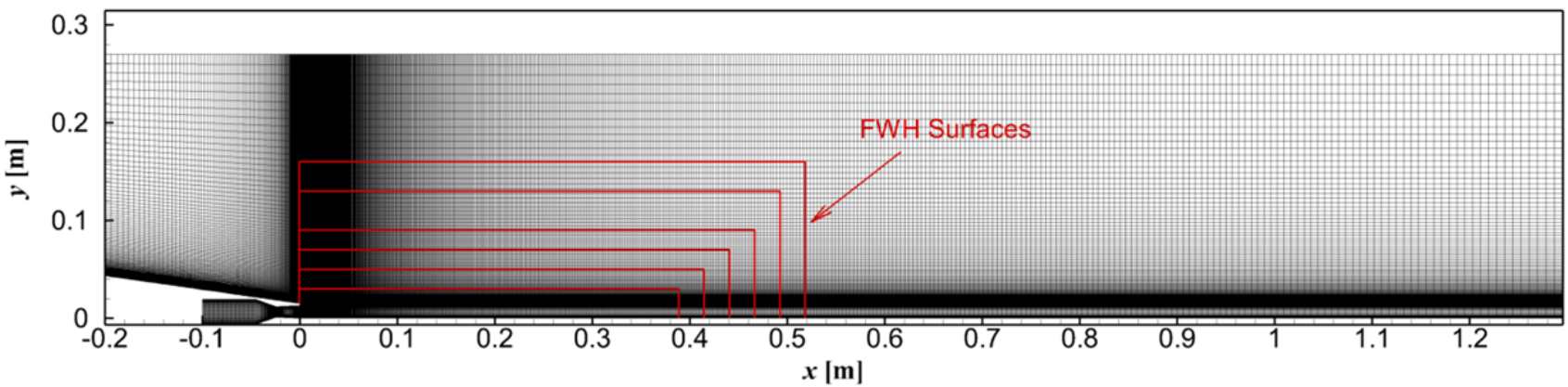

Figure 11: CFD mesh with porous FWH surfaces for jet noise prediction.

Example of Ffowcs-Williams and Hawkings Jet Noise Prediction

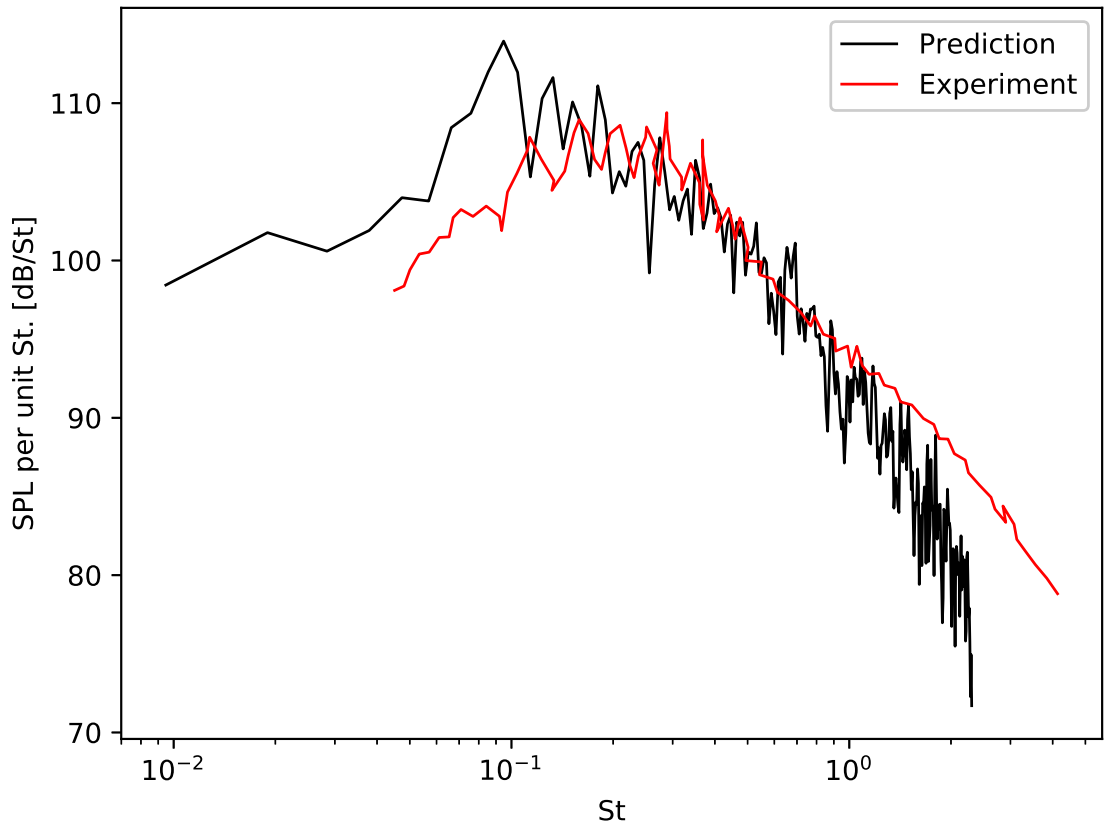

Figure 12: Comparison between prediction using LES with FWH and measurement for a supersonic jet at $\psi = 152$ deg.

Tam's Fine-Scale Model - An Alternative Approach I

Tam and Auriault [12] developed theory to predict noise from fine-scale turbulence

- Premise that jet mixing noise consists of two distinct spectra: fine-scale and large-scale
- Based on experimental observations of Tam et al. [13]

Proposed that acoustic pressure is related to k of the 'fine-scale' turbulence

$$p = (1/3)\rho < \boldsymbol{u}^2 >= (2/3)\rho K_s \tag{15}$$

- Chose linearized Euler equations as governing equations with right hand side source equal to $-\partial q_s/\partial x_i$
- Their predictions agreed extremely well with measurements that scale with M and T

Tam's Fine-Scale Model - An Alternative Approach I

- Need a model that governs the propagation with a source term
- The linearized Euler equations in cylindrical coordinates with a parallel mean flow and source are

$$\overline{\rho}\left[\frac{\partial u}{\partial t}+\overline{u}\frac{\partial u}{\partial x}+v\frac{\partial p}{\partial r}\right]=-\frac{\partial q_s}{\partial x} \tag{16}$$

$$\overline{\rho}\left[\frac{\partial v}{\partial t}+\overline{u}\frac{\partial v}{\partial x}\right]+\frac{\partial p}{\partial r}=-\frac{\partial q_s}{\partial r} \tag{17}$$

$$\overline{\rho}\left[\frac{\partial w}{\partial t}+\overline{u}\frac{\partial w}{\partial x}\right]+\frac{1}{r}\frac{\partial p}{\partial \phi}=-\frac{1}{r}\frac{q_s}{\partial \phi} \tag{18}$$

$$\frac{\partial p}{\partial t}+\overline{u}\frac{\partial p}{\partial x}+\gamma\overline{p}\left[\frac{1}{r}\frac{\partial vr}{\partial r}+\frac{1}{r}\frac{\partial w}{\partial \phi}+\frac{\partial u}{\partial x}\right]=0 \tag{19}$$

Tam's Fine-Scale Model - An Alternative Approach I

To solve the problem we must solve the vector Green's function problem

$$\overline{\rho}\left[\frac{\partial u_n}{\partial t}+\overline{u}\frac{\partial u_n}{\partial x}+v_n\frac{\partial p}{\partial r}\right]=\delta(\boldsymbol{x}-\boldsymbol{x}_1)\delta(t-t_1)\delta_{n1} \tag{20}$$

$$\overline{\rho}\left[\frac{\partial v_n}{\partial t}+\overline{u}\frac{\partial v_n}{\partial x}\right]+\frac{\partial p_n}{\partial r}=\delta(\boldsymbol{x}-\boldsymbol{x}_1)\delta(t-t_1)\delta_{n2} \tag{21}$$

$$\overline{\rho}\left[\frac{\partial w_n}{\partial t}+\overline{u}\frac{\partial w_n}{\partial x}\right]+\frac{1}{r}\frac{\partial p_n}{\partial \phi}=\delta(\boldsymbol{x}-\boldsymbol{x}_1)\delta(t-t_1)\delta_{n3} \tag{22}$$

$$\frac{\partial p_n}{\partial t}+\overline{u}\frac{\partial p_n}{\partial x}+\gamma\overline{p}\left[\frac{1}{r}\frac{\partial v_n r}{\partial r}+\frac{1}{r}\frac{\partial w_n}{\partial \phi}+\frac{\partial u_n}{\partial x}\right]=0 \tag{23}$$

Tam's Fine-Scale Model - An Alternative Approach I

Using an adjoint approach for the vector Green's function we can write the solution as

$$p(\boldsymbol{x},t) = \int\int\int\int\int \left[u_a(\boldsymbol{x}_1,\boldsymbol{x},\omega)\frac{\partial q_s(\boldsymbol{x}_1,t_1)}{\partial x_1} + v_a(\boldsymbol{x}_1,\boldsymbol{x},\omega)\frac{\partial q_s(\boldsymbol{x}_1,t_1)}{\partial r_1} + \frac{w_a(\boldsymbol{x}_1,\boldsymbol{x},\omega)}{r}\frac{\partial q_s(\boldsymbol{x}_1,t_1)}{\partial \phi}\right] \times \exp\left[-i\omega(t-t_1)\right] d\omega dt_1 d\boldsymbol{x}_1. \tag{24}$$

Statistical predictions are made by finding the spectral density, S, from Eqn. 24

$$S(\boldsymbol{x},\omega) = 4\pi\left(\frac{\pi}{\log 2}\right)^{3/2}\int\int\int \frac{\hat{q}_s^2 l_s^3}{c^2\tau_s} \times \left\{ |p_s|^2 \exp\left[\frac{-\omega^2 l_s^2}{\overline{u}^2/(4\log 2)}\right]\left[1+\omega^2\tau_s^2\left(1-\overline{u}\cos\Theta/c_\infty\right)^2\right]^{-1}\right\} d\boldsymbol{x}_2. \tag{25}$$

Tam's Fine-Scale Model - An Alternative Approach

Example prediction of Tam's fine-scale method applied to jet noise

- Valid for certain radiation directions and for the fine-scale source
- Scales with jet velocity and temperature
- What does this mean in terms of a similarity theory of noise radiation from turbulence?

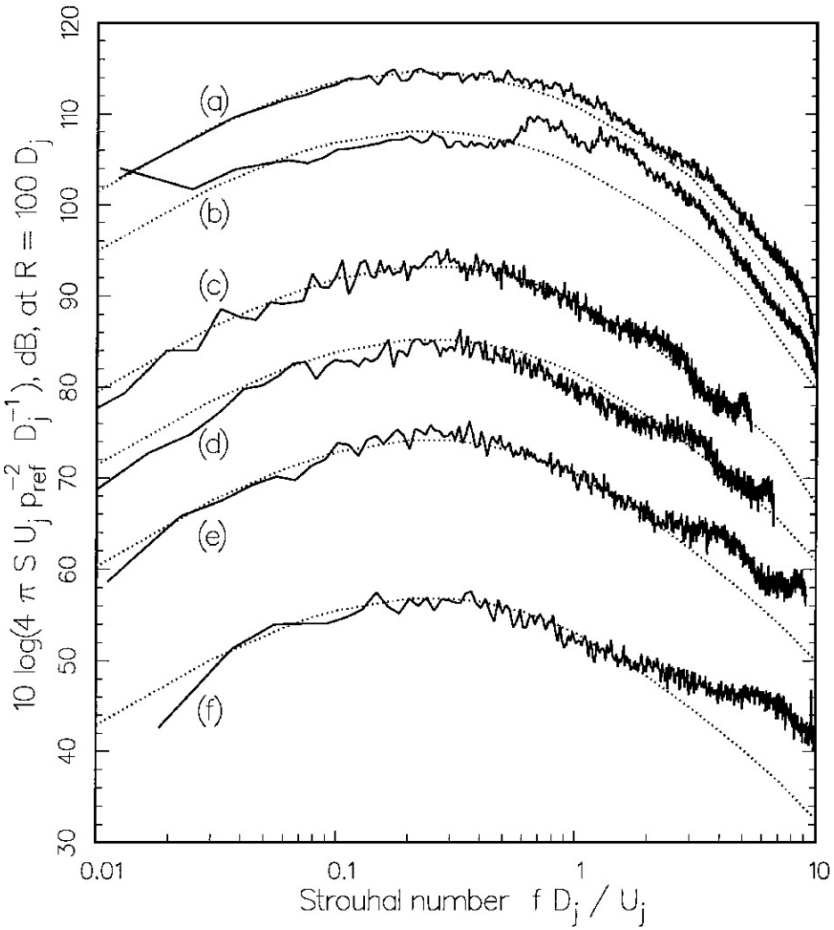

Figure 13: Predictions of Tam and Auriault [12] at $\theta = 90$ deg, TTR = 1, and various M_j.

The Impact of Tam's Approach - Two Source Model I

- Tam et al. [13] proposes noise occurs due to fine-scale and large-scale turbulence
- Proposes the empirical large-scale spectrum (LSS) and fine-scale spectrum (FSS)
- Summations of LSS and FSS match wide set of jet noise data giving credibility to the two-source model
- But turbulence is a cascade ...

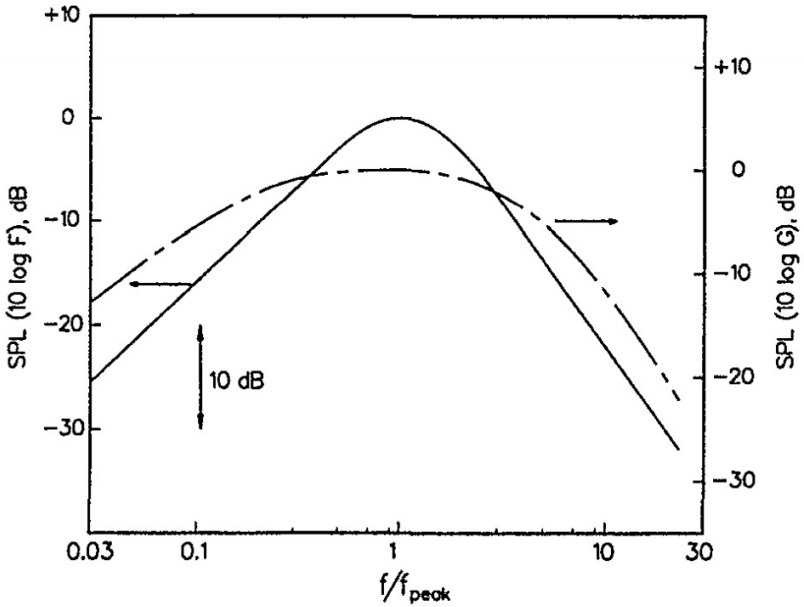

Figure 2. Similarity spectra for the two components of turbulent mixing noise. ——— large turbulence structures/ instability waves noise; — - — fine scale turbulence noise.

Figure 14: The FSS and LSS from Tam et al. [13].

The Impact of Tam's Approach - Two Source Model I

- Best fit of the fine-scale and large-scale similarity spectra
- Note similarity spectra are empirical functions

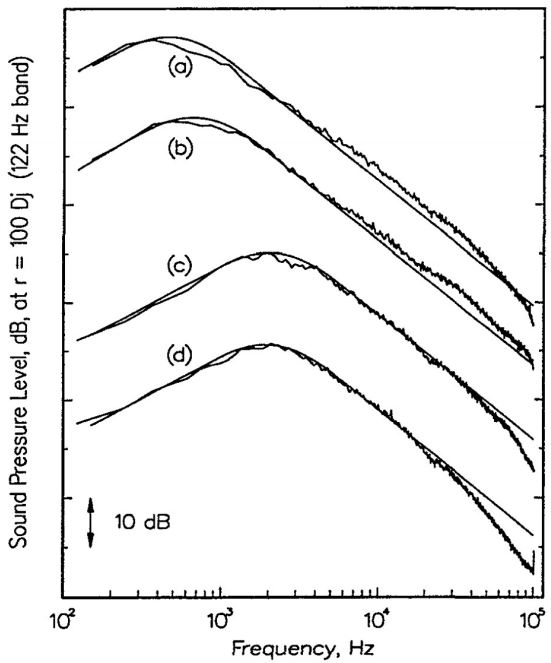

Figure 3. Comparison of the similarity spectrum of large turbulence structures/instability waves noise and measurements.
(a) $M_j = 2.0$, $T_r/T_\infty = 4.89$, $\chi = 160.1°$, $SPL_{max} = 124.7$ dB,
(b) $M_j = 2.0$, $T_r/T_\infty = 1.12$, $\chi = 160.1°$, $SPL_{max} = 121.6$ dB,
(c) $M_j = 1.96$, $T_r/T_\infty = 1.78$, $\chi = 138.6°$, $SPL_{max} = 121.0$ dB,
(d) $M_j = 1.49$, $T_r/T_\infty = 1.11$, $\chi = 138.6°$, $SPL_{max} = 106.5$ dB.

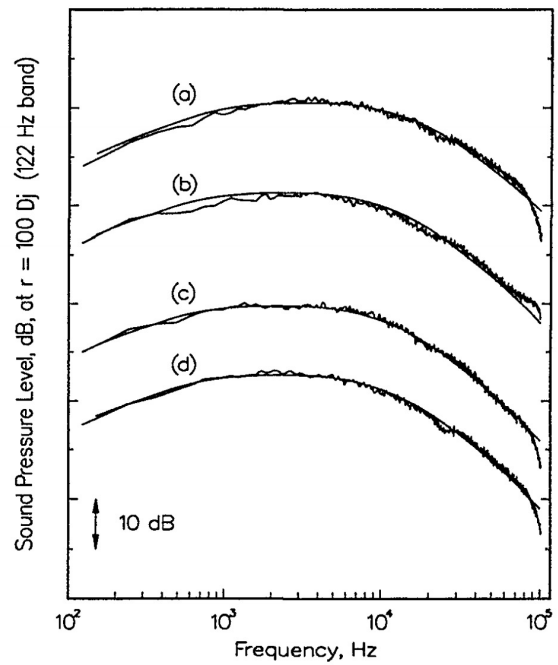

Figure 4. Comparison of the similarity spectrum of fine scale turbulence and measurements.
(a) $M_j = 1.49$, $T_r/T_\infty = 2.35$, $\chi = 92.9°$, $SPL_{max} = 96$ dB,
(b) $M_j = 2.0$, $T_r/T_\infty = 4.89$, $\chi = 83.8°$, $SPL_{max} = 107$ dB,
(c) $M_j = 1.96$, $T_r/T_\infty = 0.99$, $\chi = 83.3°$, $SPL_{max} = 95$ dB,
(d) $M_j = 1.96$, $T_r/T_\infty = 0.98$, $\chi = 120.2°$, $SPL_{max} = 100$ dB.

Figure 15: Example fit of FSS and LSS to data from Tam et al. [13].

Observations and Problems with Previous Approaches I

- Source term often has acoustic fluctuations (solution is required to be known or approximated in advance)
- Some approaches combine refraction effects with the equivalent source
- The source terms are equivalent and not the actual source
- Particular approaches do not allow insight into physics of turbulence or sound source (eg FWH)
- Majority of approaches do not allow for the prediction of the acoustic near-field
- Need a new approach that overcomes these limitations ...

A New Model

We base our approach on the Navier-Stokes equations. With continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0, \tag{26}$$

momentum

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = \frac{\partial \tau_{ij}}{\partial x_j}, \tag{27}$$

and energy

$$\frac{\partial \rho e_o}{\partial t} + \frac{\partial \rho u_j e_o}{\partial x_j} = -\frac{\partial u_j p}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + \frac{\partial u_j \tau_{ij}}{\partial x_j}, \tag{28}$$

where

$$\tau_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij}. \tag{29}$$

A New Model

We decompose the field-variables (dependent variables) as

$$q = \overline{q} + \tilde{q} + \tilde{\tilde{q}} + q' + q'', \tag{30}$$

where $\boldsymbol{q}$ is the vector of the field-variables.

- Decomposition is a base flow, fluctuations of anisotropic turbulence, fluctuations of isotropic turbulence, and respective radiated acoustic waves

Solve by convolution of the vector source term with the vector Green's function of the linearized Navier-Stokes equations

$$\boldsymbol{q}'_k(\boldsymbol{x}, t) = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \sum_{n=0}^{3} \boldsymbol{q}_{g,k,n}(\boldsymbol{x}, t; \boldsymbol{y}, \tau)\, \boldsymbol{\Theta}_n(\boldsymbol{y}, \tau)\, d\tau d\boldsymbol{y}. \tag{31}$$

Navier-Stokes Equations - The Sources

Sources on the right hand side are for mass,

$$\Theta_0 = -\frac{\partial \underline{\rho}}{\partial t} - \frac{\partial \underline{\rho}\underline{u}_j}{\partial x_j} \tag{32}$$

momentum,

$$\Theta_i = -\frac{\partial}{\partial t}\left(\underline{\rho}\underline{u}_i\right) - \frac{\partial}{\partial x_j}\left(\underline{\rho}\underline{u}_i\underline{u}_j\right) + \frac{\partial}{\partial x_j}\left\{\mu\left[\frac{\partial \underline{u}_i}{\partial x_j} + \frac{\partial \underline{u}_j}{\partial x_i}\right]\right\} - \frac{2}{3}\frac{\partial}{\partial x_j}\left\{\mu\frac{\partial}{\partial x_k}\underline{u}_k\right\} - \frac{\partial \underline{p}}{\partial x_j}\delta_{ij} \tag{33}$$

and energy,

$$\Theta_4 = -\frac{\partial \underline{p}}{\partial t} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\underline{u}_k\underline{u}_k}{\partial t} - \gamma\frac{\partial \underline{u}_j\underline{p}}{\partial x_j} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\underline{u}_j\underline{u}_k\underline{u}_k}{\partial x_j} + (\gamma-1)\frac{\partial}{\partial x_j}\left(\frac{c_p\mu}{Pr}\frac{\partial}{\partial x_j}\left[\underline{T}\right]\right)$$
$$+ (\gamma-1)\frac{\partial}{\partial x_j}\left[\mu\underline{u}_i\left(\frac{\partial \underline{u}_i}{\partial x_j} + \frac{\partial \underline{u}_j}{\partial x_i}\right)\right] - \frac{2}{3}\delta_{ij}\frac{\partial}{\partial x_j}\left[\mu\left(\underline{u}_i\right)\frac{\partial \underline{u}_k}{\partial x_k}\right] \tag{34}$$

The Vector Green's Function

The vector Green's function, $\boldsymbol{q}_{g,k,n}\left(\boldsymbol{x},t;\boldsymbol{y},\tau\right)$, must satisfy the conservation of mass

$$\frac{\partial \rho_{g,n}}{\partial t}+\frac{\partial}{\partial x_j}\left[\rho_{g,n}\overline{u}_j+u_{j,g,n}\overline{\rho}\right]=\delta\left(\boldsymbol{x}-\boldsymbol{y}\right)\delta\left(t-\tau\right)\delta_{0n}, \tag{35}$$

conservation of momentum

$$\begin{aligned}\frac{\partial}{\partial t}\left[\rho_{g,n}\overline{u}_i+u_{i,g,n}\overline{\rho}\right]+\frac{\partial}{\partial x_j}\left[\rho_{g,n}\overline{u}_i\overline{u}_j+u_{i,g,n}\overline{\rho u}_j+u_{j,g,n}\overline{\rho u}_i\right]-\overline{\mu}\frac{\partial}{\partial x_j}\left[\frac{\partial u_{i,n}}{\partial x_j}+\frac{\partial u_{j,n}}{\partial x_i}\right]\\ +\frac{2}{3}\overline{\mu}\frac{\partial^2 u_{k,n}}{\partial x_j\partial x_k}+\frac{\partial p_{g,n}}{\partial x_j}\delta_{ij}=\delta\left(\boldsymbol{x}-\boldsymbol{y}\right)\delta\left(t-\tau\right)\delta_{in},\end{aligned} \tag{36}$$

and conservation of energy

$$\begin{aligned}&\frac{\partial p_{g,n}}{\partial t}+\frac{\gamma-1}{2}\frac{\partial}{\partial t}\left[\rho_{g,n}\overline{u}_k^2+2u_{k,g,n}\overline{\rho u}_k\right]+\gamma\frac{\partial}{\partial x_j}\left[p_{g,n}\overline{u}_j+u_{j,g,n}\overline{p}\right]\\ &+\frac{\gamma-1}{2}\frac{\partial}{\partial x_j}\left[\rho_{g,n}\overline{u}_j\overline{u}_k^2+u_{j,g,n}\overline{\rho u}_k^2+2u_{k,g,n}\overline{\rho u}_k\right]-\frac{\partial}{\partial x_j}\frac{c_p\mu}{\mathcal{P}rR}\frac{\partial}{\partial x_j}\left[\frac{\tilde{p}_{g,n}}{\tilde{\rho}_{g,n}}\right]\\ &-\left(\gamma-1\right)\overline{\mu}\frac{\partial}{\partial x_j}\left[\overline{u}_i\left(\frac{\partial u_{i,g,n}}{\partial x_j}+\frac{u_{j,g,n}}{\partial x_i}\right)+u_{i,g,n}\left(\frac{\partial\overline{u}_i}{\partial x_j}+\frac{\partial\overline{u}_j}{\partial x_i}\right)\right]\\ &+\frac{2}{3}\delta_{ij}\frac{\partial}{\partial x_j}\left[\mu\left(\overline{u}_i\right)\frac{\partial u_{k,g,n}}{\partial x_k}+\mu\left(u_{i,g,n}\right)\frac{\partial\overline{u}_k}{\partial x_k}\right]\\ &=\delta\left(\boldsymbol{x}-\boldsymbol{y}\right)\delta\left(t-\tau\right)\delta_{3n}.\end{aligned} \tag{37}$$

Forming the Spectral Density and Source Correlation

We define the spectral density, S_k, as the inverse Fourier transform of the auto-correlation

$$S_k(\boldsymbol{x},\omega) = \int_{-\infty}^{\infty} \left\langle q_k(\boldsymbol{x},t)\, q_k\left(\boldsymbol{x},t+\tau^{\dagger}\right)\right\rangle \exp\left[i\omega\tau^{\dagger}\right] d\tau^{\dagger}. \tag{38}$$

Let $R_{m,n}(\boldsymbol{y},\boldsymbol{\eta},\tau)$ be the two-point space-time cross-correlation of the equivalent source

$$\begin{aligned} R_{m,n}(\boldsymbol{y},\boldsymbol{\eta},\tau) &= \langle \Theta_m(\boldsymbol{y},\tau)\,\Theta_n(\boldsymbol{y}+\boldsymbol{\eta},\tau+\tau)\rangle \\ &= \int_{-\infty}^{\infty} \Theta_m(\boldsymbol{y},\tau)\,\Theta_n(\boldsymbol{y}+\boldsymbol{\eta},\tau+\tau)\, d\tau, \end{aligned} \tag{39}$$

where $\langle\rangle$ is the two-point space-time cross-correlation operator.

A Prediction Equation for Spectral Density

Using this definition and Eqn. 39, we obtain the k^{th} spectral density

$$S_k(\boldsymbol{x},\omega) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \sum_{m=0}^{3} \sum_{n=0}^{3} q^*_{g,k,m}(\boldsymbol{x};\boldsymbol{y},\omega)\, q_{g,k,n}(\boldsymbol{x};\boldsymbol{y}+\boldsymbol{\eta},\omega) \times R_{m,n}(\boldsymbol{y},\boldsymbol{\eta},\tau)\, d\tau d\boldsymbol{\eta} d\boldsymbol{y}, \tag{40}$$

which is the prediction equation for spectral density of pressure ($k = 4$).

One of Many Two-Point Correlations (sources) of the NSE Terms

The first term of $R^{\perp}_{m,n}$ (of many) for a steady baseflow is

$$\left\langle \frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial \underline{\rho}^{(2)} \underline{u}_m^{(2)}}{\partial y_m} \right\rangle =$$

$$\frac{\partial \overline{\rho}^{(2)} \overline{u}_m^{(2)}}{\partial y_m} \frac{\hat{\rho}^{(1)}}{\hat{\tau}_s^{(1)}} \hat{R} + \frac{\hat{\rho}^{(1)}}{\hat{\tau}_s^{(1)}} \left(\overline{\rho}^{(2)} \frac{\hat{u}_m^{(2)}}{l_{m,s}} + \frac{\partial \overline{\rho}^{(2)}}{\partial y_m} \hat{u}_m^{(2)} \right) \hat{R} + \text{... additional terms expanded ...}$$

$$+ \frac{\breve{\rho}^{(1)}}{\breve{\tau}_s} \left(\breve{\rho}^{(2)} \frac{\partial \overline{u}_m^{(2)}}{\partial y_m} + \overline{u}_m^{(2)} \frac{\breve{\rho}^{(2)}}{\breve{l}_s} \right) \hat{\breve{R}} + \frac{\breve{\rho}^{(1)}}{\breve{\tau}_s} \left(\frac{\breve{\rho}^{(2)} \hat{u}_m^{(2)}}{\hat{l}_s} + \frac{\breve{\rho}^{(2)} \hat{u}_m^{(2)}}{\breve{l}_s} \right) \hat{\breve{R}} + \frac{\breve{\rho}^{(1)} \breve{\rho}^{(2)} \breve{u}_m^{(2)}}{\breve{\tau}_s \breve{l}_s} \breve{R} \tag{41}$$

where the hat and breve symbols are for anisotropic turbulence and isotropic turbulence, respectively.

- We introduced three types of normalized two-point cross-correlations $\hat{R}$, $\breve{R}$, and $\hat{\breve{R}}$.

Modeling the Small Scale Spatially Incoherent Turbulence

We relate the energy at κ to the energy spectrum by

$$\breve{u}_i\left(\boldsymbol{y},\tau\right)=\sqrt{\frac{2}{3}}\left(\int_{\kappa_1}^{\kappa_2}E_u(\boldsymbol{y},\kappa,\tau)d\kappa\right)^{1/2}. \tag{42}$$

Also, the spectra of temperature, E_T, is

$$\breve{T}\left(\boldsymbol{y},\tau\right)=\left(\int_{\kappa_1}^{\kappa_2}E_T(\boldsymbol{y},\kappa,\tau)d\kappa\right)^{1/2}. \tag{43}$$

Other spectra are defined similarly for other field-variables ρ and p

Results - Spectrum of Two-Point Structure Function of Jet Turbulence

- LES provides spectra of field-variables
- Extract portion of energy due to anisotropic and isotropic components
- Extract meanflow
- Use extracted components within acoustic analogy

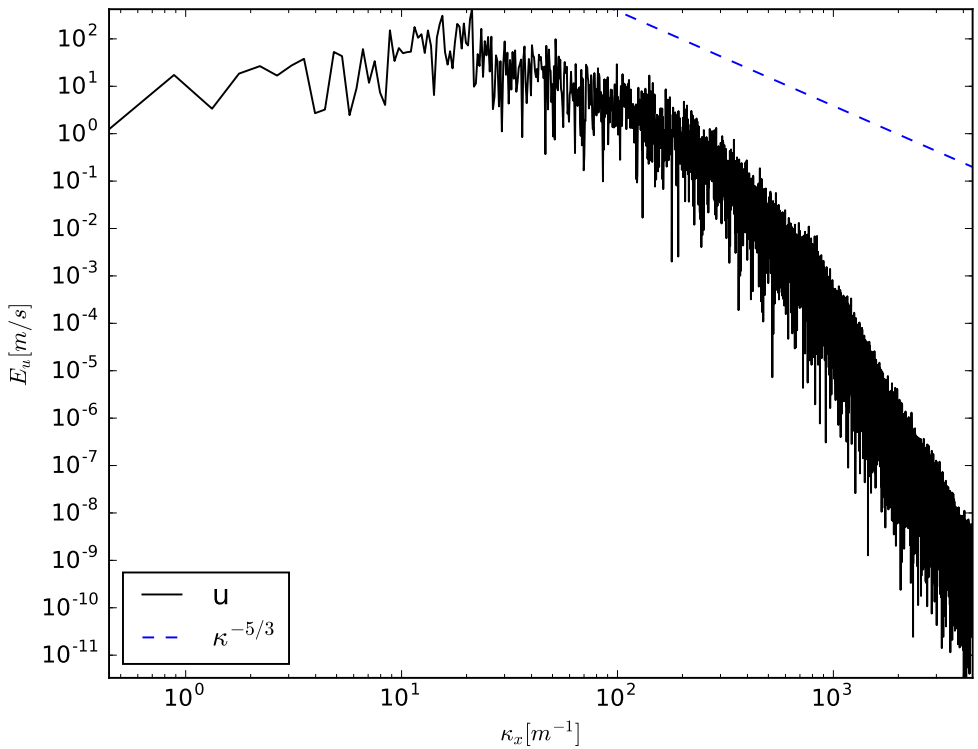

Figure 16: Spectrum of *u* within a shear layer.

Results - Velocity Spectrum Decomposition

An example of a decomposition of an energy spectra

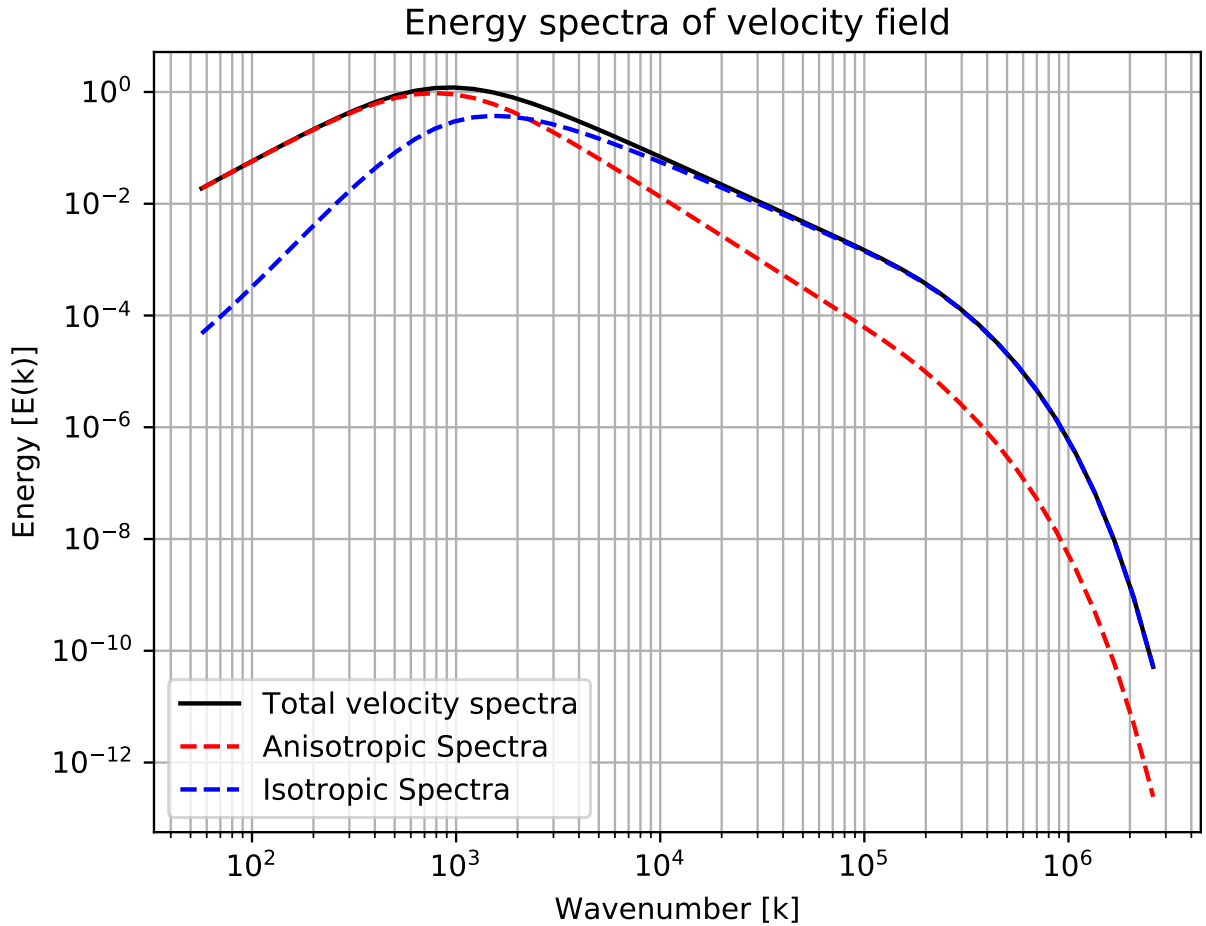

Figure 17: Sample decomposition of velocity spectra at spatial point within turbulent jet flow. Courtesy of Trushant Patel (UF MAE Ph.D. student).

Results - Small-Scale Incoherent Structures

Predictions of noise from isotropic turbulent field

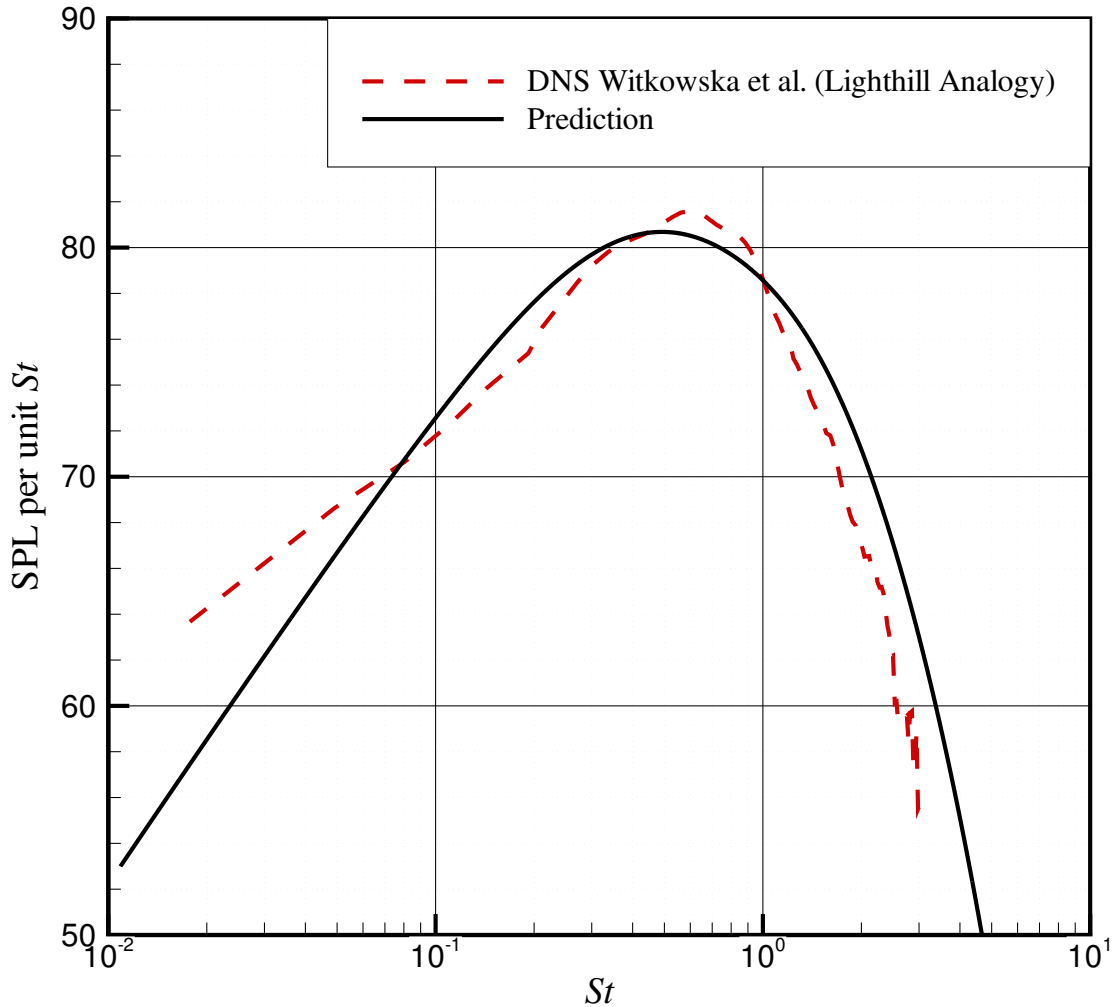

Figure 18: SPL prediction compared with DNS of Witkowska.

Results - Large-Scale Coherent Structures

Predictions of noise from anisotropic turbulent jet

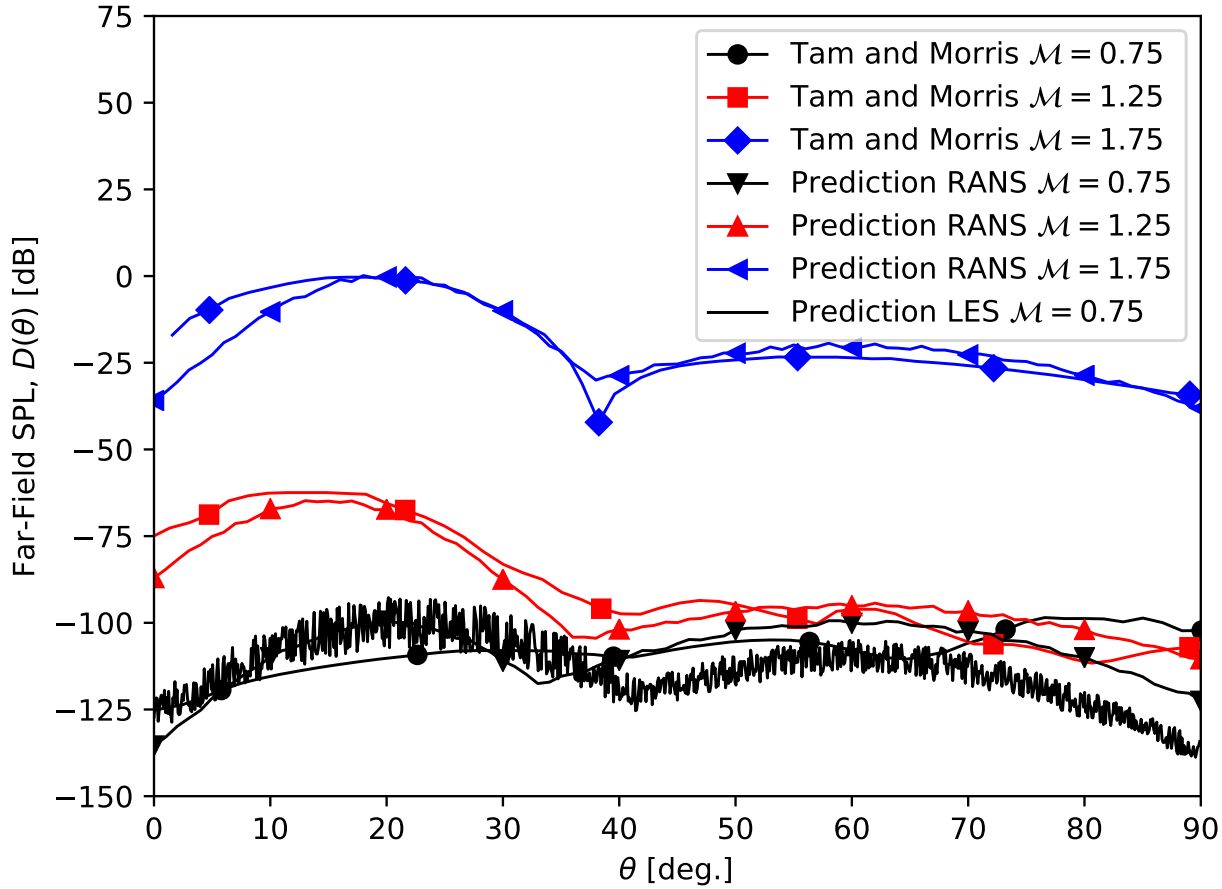

Figure 19: SPL prediction compared to large-scale turbulent noise predictions of Tam and Morris.

Advantages of the New Approach

- Sources are the Navier-Stokes equations (NSE) without acoustic fluctuations
- Two-point cross-correlations (sources) are between each NSE term
- Vector Green's function, q_g^n, are for the linearized NSE
- Analytical Green's function is known for quiescent meanflow but must be found numerically for most non-zero-meanflows
- Source modeling is divided into highly coherent large-scale turbulence and incoherent small-scale turbulence
- Source model is directly analogous to Tam's FSS (fine-scale) and LSS (large-scale) models, but more general because it involves the entire turbulent cascade

Conclusion

- A new approach is being developed that resolves outstanding technical issues
- Contemporary acoustic analogies are emerging and their philosophy is highly debated among theoreticians
- The field has a rich history with important scientific and social implications
- Performing research in this field is time-consuming, requires substantial knowledge in mathematics, and the willingness to pursue turbulent theories

Thank You

Questions?

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