Decomposition of the Navier-Stokes Equations for Noise Source Quantification within Turbulent Jets

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Motivation

- Jet noise remains a community annoyance and a major source of hearing damage for military personnel
- The physical and mathematical understanding of the noise sources are critical for the purpose of reduction
- Identify source terms in context of Navier-Stokes equations for noise reduction

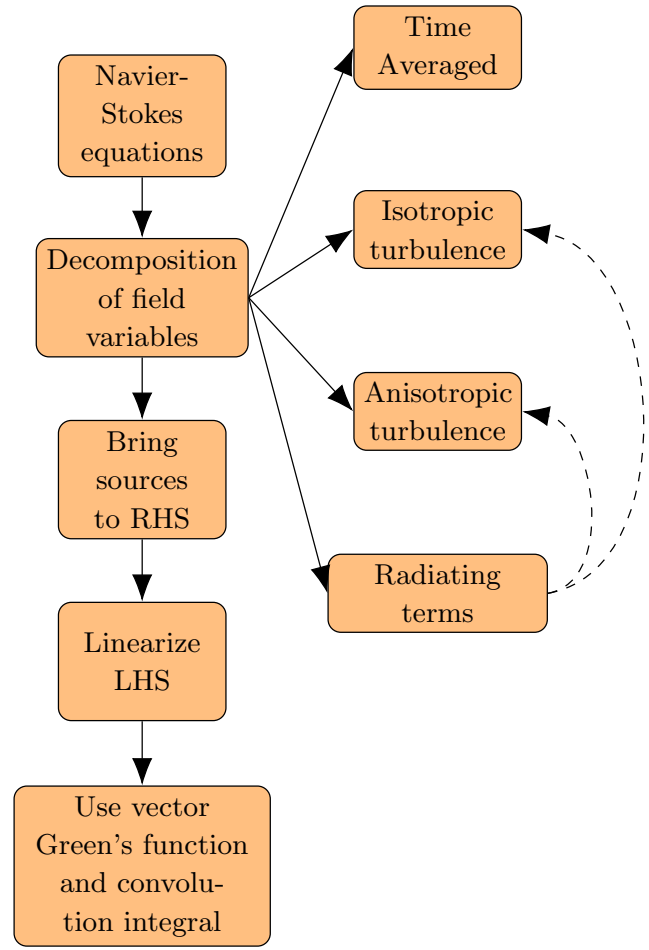
Figure 1: F/A-18C Hornet on USS Theodore Roosevelt. US Navy public release photo: 180123-N-GP724-1102.

Objectives

- Identify source terms via theory of two-point correlations of Navier-Stokes equations decomposition
- Decomposition consists of large-scale spatially coherent structures, small-scale relatively incoherent structures, and associated radiated noise components
  - Example shown in this presentation – broadband shock-associated noise (BBSAN)
- Perform large-eddy simulation (LES) of supersonic jet, validate LES, and decompose with proper orthogonal decomposition (POD)
  - POD is used to identify large-scale turbulent structures
- Use LES POD to evaluate two-point sources to validate source terms identified from Navier-Stokes theory

Statistical Solution for Acoustics via Navier-Stokes

- Field-variables consisting of density, velocity, pressure, and temperature are decomposed into base flow, aerodynamic fluctuations, and acoustic fluctuations
- Radiating acoustic components (propagators) are kept on the left-hand side
- Aerodynamic turbulent fluctuations (sources) are brought to the right-hand side
- Spectral density of the radiating components is obtained by convolving the vector Green's function with the source terms present on the right-hand side



Decomposition of the Field-Variables

- Decomposition of field variables

$$q = \overline{q} + \breve{q} + \hat{q} + q' + q''$$

$\overline{q}$ = Base quantity
$\breve{q}$ = Turbulent fluctuations from spatially incoherent isotropic structures
$\hat{q}$ = Turbulent fluctuations from spatially coherent anisotropic structures
q' = Acoustic radiation from anisotropic structures
q'' = Acoustic radiation from isotropic structures

- Substitute this decomposition into Navier-Stokes equations
- Resulting equations are rearranged and source terms are brought to the right-hand side
- Acoustic fluctuations are kept on the left-hand side

Aerodynamic Source Terms

The source terms on the right-hand side are

$$\Theta_0 = -\frac{\partial \underline{\rho}}{\partial t} - \frac{\partial \underline{\rho}\underline{u}_j}{\partial x_j},$$

$$\Theta_i = -\frac{\partial \underline{\rho}\underline{u}_i}{\partial t} - \frac{\partial \underline{\rho}\underline{u}_i\underline{u}_j}{\partial x_j} - \frac{\partial \underline{p}}{\partial x_j}\delta_{ij} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \underline{u}_i}{\partial x_j} + \frac{\partial \underline{u}_j}{\partial x_i}\right)\right] - \frac{2}{3}\frac{\partial}{\partial x_j}\left[\mu\frac{\partial \underline{u}_k}{\partial x_k}\right],$$

and

$$\Theta_4 = -\frac{\partial \underline{p}}{\partial t} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\underline{u}_k\underline{u}_k}{\partial t} - \gamma\frac{\partial \underline{u}_j\underline{p}}{\partial x_j} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\underline{u}_j\underline{u}_k\underline{u}_k}{\partial x_j} + (\gamma-1)\frac{\partial}{\partial x_j}\left[\frac{c_p\mu}{Pr}\frac{\partial \underline{T}}{\partial x_j}\right]$$
$$+(\gamma-1)\frac{\partial}{\partial x_j}\left[\mu\underline{u}_i\left(\frac{\partial \underline{u}_i}{\partial x_j} + \frac{\partial \underline{u}_j}{\partial x_i}\right)\right] - \frac{2}{3}\delta_{ij}\frac{\partial}{\partial x_j}\left[\mu\underline{u}_i\frac{\partial \underline{u}_k}{\partial x_k}\right].$$

The sources are the exact Navier-Stokes equations operating on $\underline{q} = \overline{q} + \hat{q} + \breve{q}$

BBSAN Source Terms

Potential BBSAN source terms

$$R^{(1)} = \left\langle \frac{\partial \underline{\rho}^{(1)} \underline{u}_i^{(1)} \underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial \underline{\rho}^{(2)} \underline{u}_l^{(2)} \underline{u}_m^{(2)}}{\partial y_m} \right\rangle,$$

$$R^{(2)} = \gamma \left\langle \frac{\partial \underline{\rho}^{(1)} \underline{u}_i^{(1)} \underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial \underline{u}_m^{(2)} \underline{p}^{(2)}}{\partial y_m} \right\rangle,$$

$$R^{(3)} = \frac{\gamma - 1}{2} \left\langle \frac{\partial \underline{\rho}^{(1)} \underline{u}_i^{(1)} \underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial \underline{\rho}^{(2)} \underline{u}_m^{(2)} \underline{u}_n^{(2)} \underline{u}_n^{(2)}}{\partial y_m} \right\rangle,$$

$$R^{(4)} = \boxed{\gamma^2 \left\langle \frac{\partial \underline{u}_j^{(1)} \underline{p}^{(1)}}{\partial y_j}, \frac{\partial \underline{u}_m^{(2)} \underline{p}^{(2)}}{\partial y_m} \right\rangle},$$

$$R^{(5)} = \frac{\gamma(\gamma - 1)}{2} \left\langle \frac{\partial \underline{u}_j^{(1)} \underline{p}^{(1)}}{\partial y_j}, \frac{\partial \underline{\rho}^{(2)} \underline{u}_m^{(2)} \underline{u}_n^{(2)} \underline{u}_n^{(2)}}{\partial y_m} \right\rangle,$$

$$R^{(6)} = \frac{(\gamma - 1)^2}{4} \left\langle \frac{\partial \underline{\rho}^{(1)} \underline{u}_j^{(1)} \underline{u}_k^{(1)} \underline{u}_k^{(1)}}{\partial y_j}, \frac{\partial \underline{\rho}^{(2)} \underline{u}_m^{(2)} \underline{u}_n^{(2)} \underline{u}_n^{(2)}}{\partial y_m} \right\rangle.$$

Ribner's Model

$$R^{(1)} = \frac{\partial \overline{\rho}^{(1)} \overline{u}_i^{(1)}}{\partial y_j} \frac{\partial \overline{\rho}^{(2)} \overline{u}_l^{(2)}}{\partial y_m} \hat{u}_j^{(1)} \hat{u}_m^{(2)} \hat{R},$$

$$R^{(2)} = \gamma \frac{\partial \overline{\rho}^{(1)} \overline{u}_i^{(1)}}{\partial y_j} \frac{\partial \overline{p}^{(2)}}{\partial y_m} \hat{u}_j^{(1)} \hat{u}_m^{(2)} \hat{R},$$

$$R^{(3)} = \frac{\gamma - 1}{2} \frac{\partial \overline{\rho}^{(1)} \overline{u}_i^{(1)}}{\partial y_j} \frac{\partial \overline{\rho}^{(2)} \overline{u}_n^{(2)} \overline{u}_n^{(2)}}{\partial y_m} \hat{u}_j^{(1)} \hat{u}_m^{(2)} \hat{R},$$

$$R^{(4)} = \boxed{\gamma^2 \frac{\partial \overline{p}^{(1)}}{\partial y_j} \frac{\partial \overline{p}^{(2)}}{\partial y_m} \hat{u}_j^{(1)} \hat{u}_m^{(2)} \hat{R}},$$

$$R^{(5)} = \frac{\gamma(\gamma - 1)}{2} \frac{\partial \overline{p}^{(1)}}{\partial y_j} \frac{\partial \overline{\rho}^{(2)} \overline{u}_n^{(2)} \overline{u}_n^{(2)}}{\partial y_m} \hat{u}_j^{(1)} \hat{u}_m^{(2)} \hat{R},$$

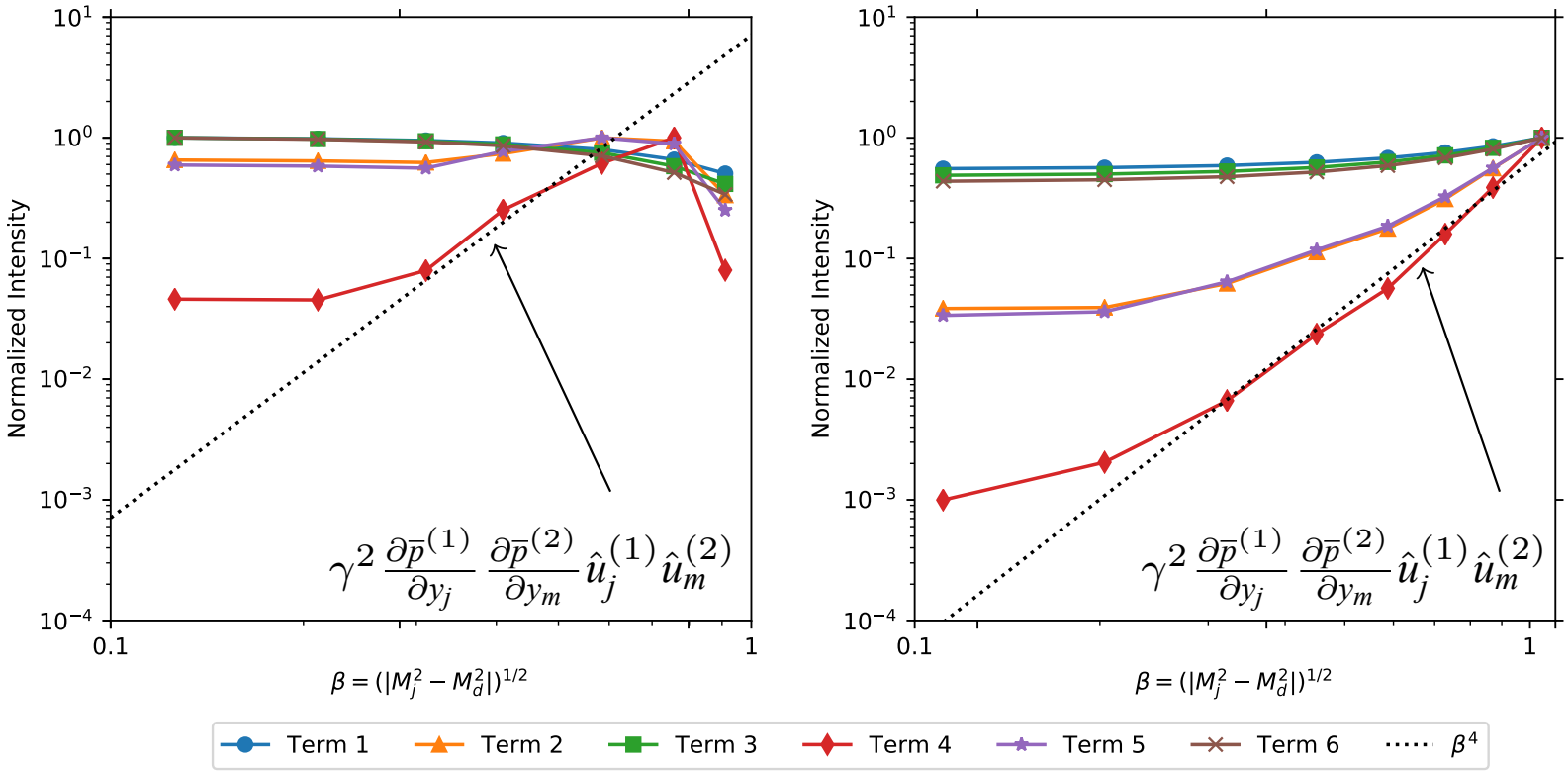
$$R^{(6)} = \frac{(\gamma - 1)^2}{4} \frac{\partial \overline{\rho}^{(1)} \overline{u}_k^{(1)} \overline{u}_k^{(1)}}{\partial y_j} \frac{\partial \overline{\rho}^{(2)} \overline{u}_n^{(2)} \overline{u}_n^{(2)}}{\partial y_m} \hat{u}_j^{(1)} \hat{u}_m^{(2)} \hat{R}.$$

- Only term that scales as β^4 is Term 4 i.e. the product of gradient of pressure and the anisotropic velocity
- Terms 1, 3, & 6 are independent of β. Term 2 & 5 scale as β^2

Scaling of BBSAN Source Term

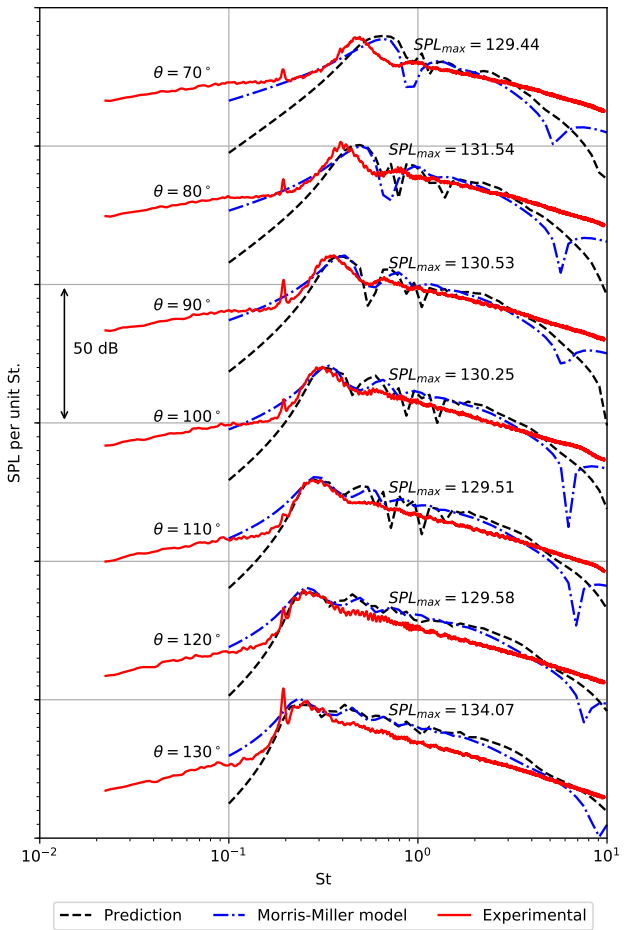
Over-expanded conditions

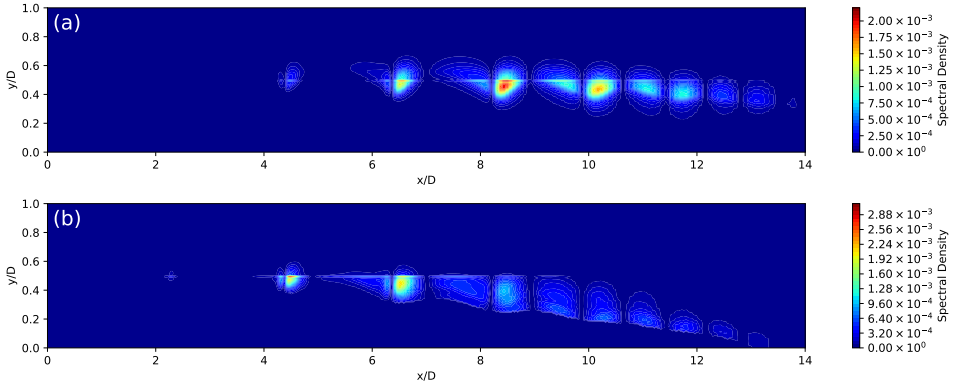
Under-expanded conditions

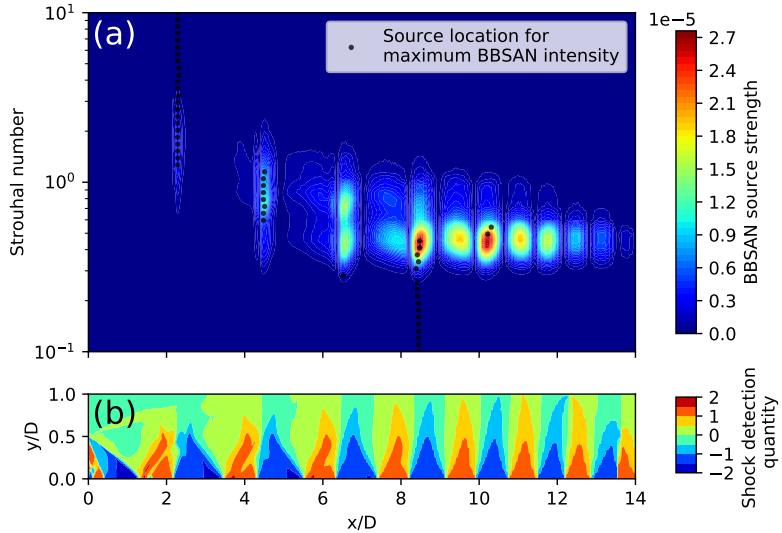


The only term that scales with BBSAN is term 4 (red diamond)

Validation and Source Location of BBSAN Term



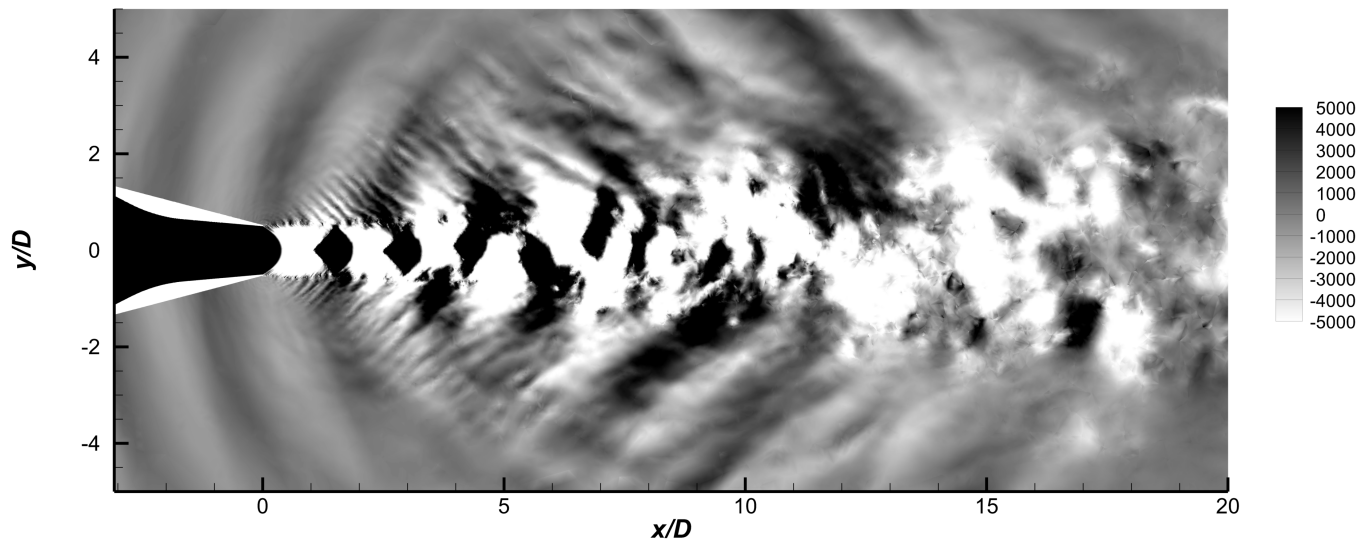




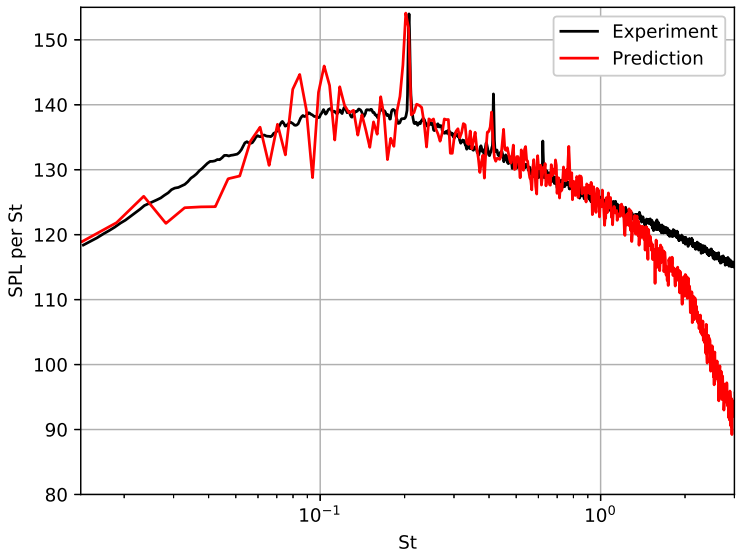
Clock–wise starting at left: Far-field predictions, acoustic source map within the jet plume, and maximum source positions. The jet operates at $M_d = 1.50$, $M_j = 1.83$, and TTR = 1.00.

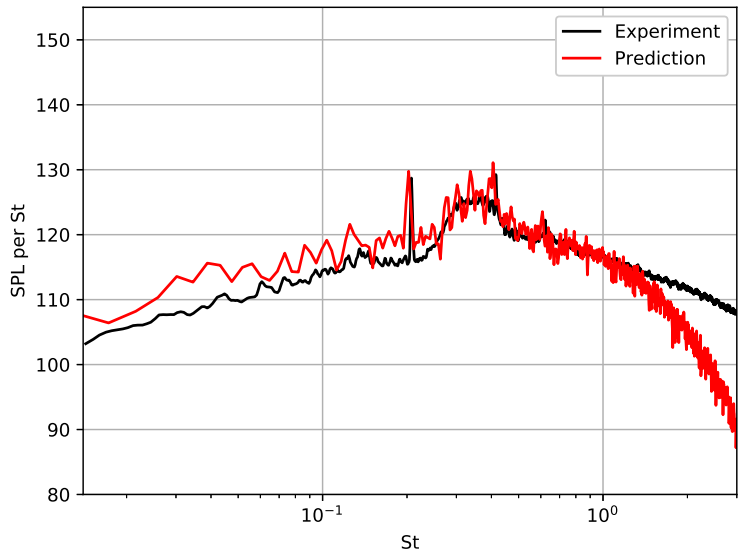
Computational Fluid Dynamics – Large-Eddy Simulation

- Compressible Navier-Stokes solver
- Energy-stable flux reconstruction scheme
- Supports unstructured hybrid mesh
- Multiple element types (tri, quad, tet, pris, hex)
- Explicit time stepping – low storage Runge-Kutta 4/5 step
- Parallelization through Message Passing Interface
- Wall adapting local eddy-viscosity model



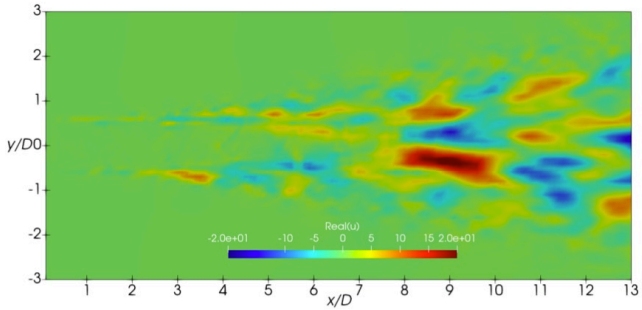
LES Validation via FWH



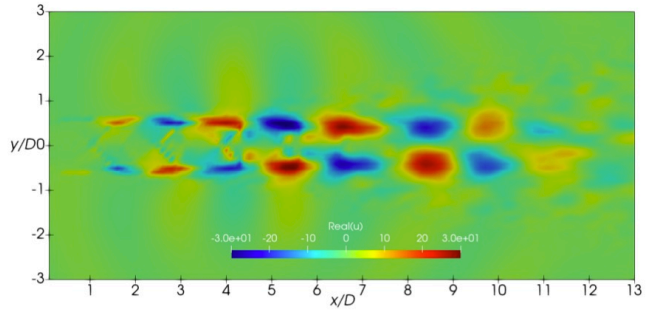


Jet conditions are NPR = 3.514, TTR = 3.2, $M_j = 1.47$, $u_j = 762.39$ m/s, $T_j = 670.4$ K, and $Re_D = 6.26 \times 10^5$.

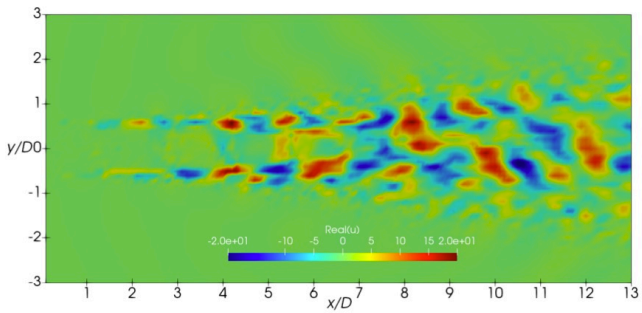
POD of LES – Contours of Velocity



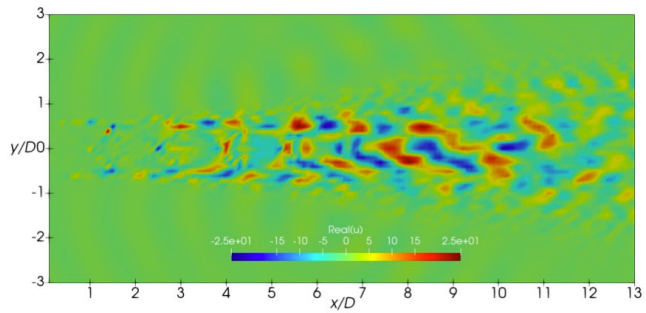
(a) $St = 0.095$.



(b) $St = 0.213$.



(c) $St = 0.308$.

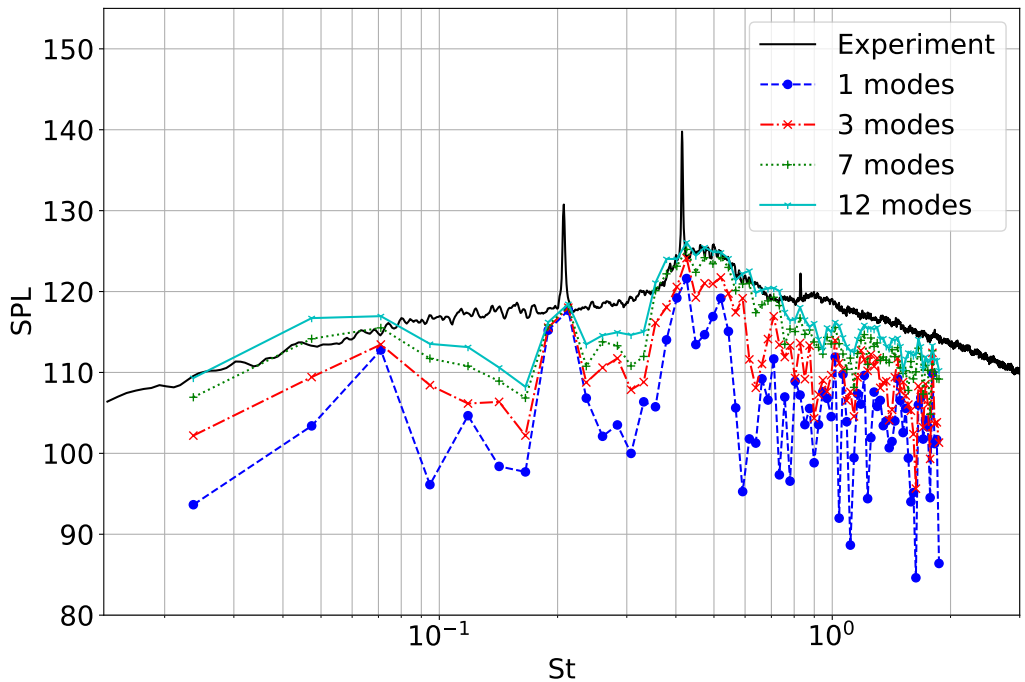


(d) $St = 0.402$.

Contours of the real part of the axial component of the velocity POD mode. Jet conditions are NPR = 3.514, TTR = 3.2, $M_j = 1.47$, $u_j = 762.39$ m/s, $T_j = 670.4$ K, and $Re_D = 6.26 \times 10^5$.

POD of LES with Present Acoustic Analogy & BBSAN Source

Lighthill's analogy with BBSAN source term



Jet conditions are NPR = 3.514, TTR = 3.2, $M_j = 1.47$, $u_j = 762.39$ m/s, $T_j = 670.4$ K, and $Re_D = 6.26 \times 10^5$.

Summary, Conclusion, and Future Research

- Created acoustic source identification method and validated BBSAN source with LES
- Major advantage is ease of quantifying both the shock-associated noise and the fine-scale mixing noise sources
- BBSAN source term is

$$\gamma^2 \left\langle \frac{\partial \underline{u}_j^{(1)} \underline{p}^{(1)}}{\partial y_j}, \frac{\partial \underline{u}_m^{(2)} \underline{p}^{(2)}}{\partial y_m} \right\rangle$$

- Future work
  - Sensitivity of BBSAN term for noise reduction – AIAA/SciTech
  - Fine-scale noise source – paper in preparation
  - Large-scale noise source – current research focus

Acknowledgements

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Thank You

Questions?

Extra Slides

Abstract

Jet noise remains a community annoyance and a major source of hearing damage for military personnel. The physical and mathematical understanding of the noise source are critical for the purpose of its reduction. We decompose the flow-field into a base flow, large-scale spatially coherent structures, small-scale relatively incoherent structures, and associated radiated noise components. The large-scale and fine-scale structures are ascertained via an additional numerical decomposition approach. An acoustic analogy based model is used to predict the noise from the relatively small-scale incoherent turbulence and the large-scale coherent turbulence interacting with the shock-cell structure. We examine these source models combined with the decomposition of large-eddy simulation (LES). We show validated noise predictions of our LES solver with both the Ffowcs Williams and Hawkings equation and our newly developed jet noise model. We evaluate the source models obtained from the acoustic analogy, that are the two-point cross-correlations of the Navier-Stokes equations, to quantify all noise sources. The major advantage of the developed approach is the ease of quantifying both the shock-associated noise and the fine-scale mixing noise sources. Finally, we discuss how our model can be used with existing LES of turbulent flows for noise reduction.

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