Herbert Wertheim College of Engineering
UNIVERSITY of FLORIDA

Navier-Stokes Equations based Acoustic Analogy with Shock-Noise Prediction Example

Trushant Patel | Steven A. E. Miller

Theoretical Fluid Dynamics and Turbulence Laboratory
Mechanical and Aerospace Engineering Department
University of Florida

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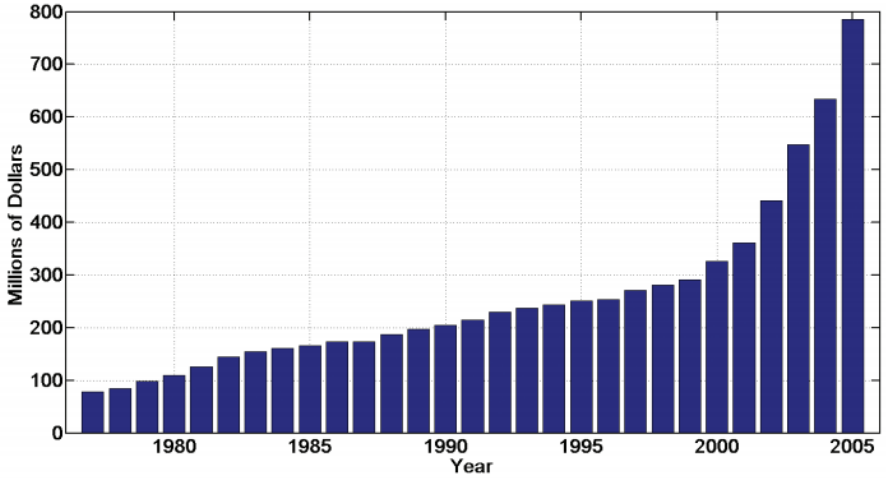
Outline

- Motivation and background.
- New acoustic analogy based on Navier-Stokes equations.
- Cross-correlation source term for shock noise.
- Results.
- Summary and Future Work.

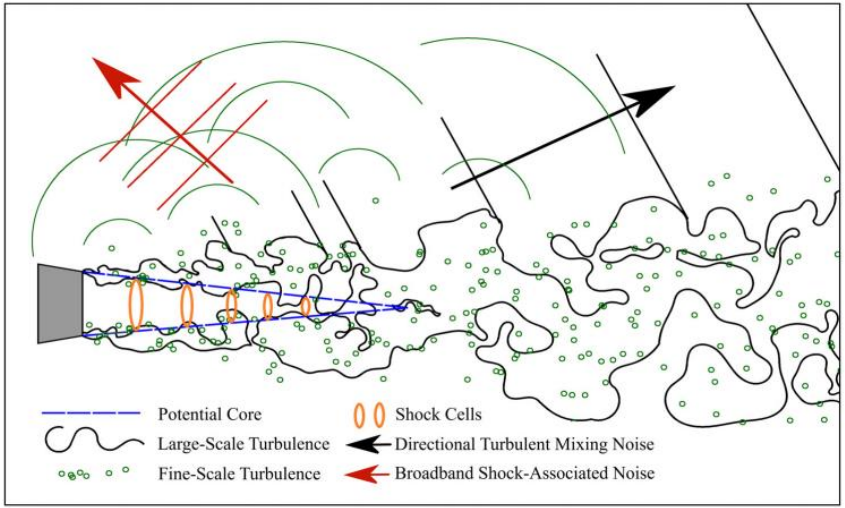
Motivation and Background

~107,000 sailors living and working aboard US Navy ships.

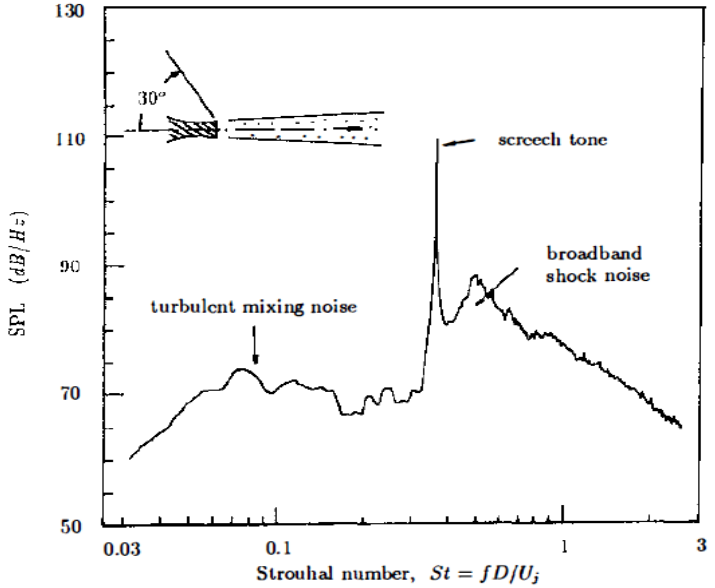


Gee, Kent L., et al. "The 'sound of freedom': Characterizing jet noise from high-performance military aircraft." *Acoustics Today* 9.3 (2013): 8-21.
http://www.public.navy.mil/navsafecen/pages/acquisition/noise_control.aspx

Motivation and Background



A. B. Vaughn, T. B. Neilsen, K. L. Gee, A. T. Wall, J. Micah Downing, and M. M. James, "Broadband shock-associated noise from a high-performance military aircraft," *The Journal of the Acoustical Society of America*, vol. 144, no. 3, pp. EL242–EL247, Sep 2018.



Tam, C. K. (1995). Supersonic jet noise. *Annual Review of Fluid Mechanics*, *27*(1), 17-43.

New Acoustic Analogy Based on N-S Equations

Navier - Stokes equations :

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_j}\delta_{ij} + \frac{\partial \tau_{ij}}{\partial x_j}$$

$$\frac{\partial \rho e_0}{\partial t} + \frac{\partial \rho u_j e_0}{\partial x_j} = -\frac{\partial u_j p}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + \frac{\partial u_j \tau_{ij}}{\partial x_j}$$

We decompose the field variables (q) from the N-S equations to:

$$q = \overline{q} + \hat{q} + \breve{q} + q' + q''$$

$\overline{q}$ = Time averaged base flow.

$\hat{q}$ = Large coherent anisotropic fluctuations.

$\breve{q}$ = Small incoherent isotropic fluctuations.

q' = Radiating waves due to anisotropic turbulence.

q'' = Radiating waves due to isotropic turbulence.

Miller, Steven A. E. "Noise from isotropic turbulence." *AIAA Journal* 55.3 (2016): 755-773.

New Acoustic Analogy Based on N-S Equations

- Bring the time averaged and fluctuating quantities to the right side, and radiating quantities to the left.
- Right side would contain source terms obtained from CFD simulation.

$$\Theta_0 = -\frac{\partial \underline{\rho}}{\partial t} - \frac{\partial \underline{\rho}}{\partial x_j} \qquad \Theta_i = -\frac{\partial \underline{\rho}\,\underline{u_i}}{\partial t} - \frac{\partial \underline{\rho}\,\underline{u_i}\,\underline{u_j}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\mu\left[\frac{\partial \underline{u_i}}{\partial x_j} + \frac{\partial \underline{u_j}}{\partial x_i}\right]\right) - \frac{2}{3}\frac{\partial}{\partial x_j}\left(\mu\frac{\partial \underline{u_k}}{\partial x_k}\right) - \frac{\partial \underline{p}}{\partial x_j}\delta_{ij}$$

$$\Theta_4 = -\frac{\partial \underline{p}}{\partial t} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\,\underline{u_k}\,\underline{u_k}}{\partial t} - \gamma\frac{\partial \underline{u_j}\,\underline{p}}{\partial x_j} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\,\underline{u_j}\,\underline{u_k}\,\underline{u_k}}{\partial x_j} + (\gamma-1)\frac{\partial}{\partial x_j}\left(\frac{c_p\mu}{Pr}\frac{\partial}{\partial x_j}\underline{T}\right)$$
$$+(\gamma-1)\frac{\partial}{\partial x_j}\left[\mu\underline{u_i}\left(\frac{\partial \underline{u_i}}{\partial x_j} + \frac{\partial \underline{u_j}}{\partial x_i}\right)\right] - \frac{2}{3}\delta_{ij}\frac{\partial}{\partial x_j}\left[\mu\underline{u_i}\frac{\partial\,\underline{u_k}}{\partial x_k}\right]$$

The sources are exact N-S equations operating on $\underline{q} = \overline{q} + \hat{q} + \breve{q}$.

Miller, Steven A. E. "Noise from isotropic turbulence." *AIAA Journal* 55.3 (2016): 755-773.

New Acoustic Analogy Based on N-S Equations

- Using vector Green's function, solution takes the form

$$q_k^{\perp}(\boldsymbol{x},t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \sum_{j=0}^{4} q_{g,k}^{\perp,j}(\boldsymbol{x},t;\boldsymbol{y},\tau)\Theta_j(\boldsymbol{y},\tau)d\tau d\boldsymbol{y}$$

where $q_k^{\perp} = q' + q''$. After some work, spectral density for pressure can be found out as,

$$S_4^{\perp}(\boldsymbol{x},\omega) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \sum_{m=0}^{4}\sum_{n=0}^{4} p_g^{*\perp,m}(\boldsymbol{x};\boldsymbol{y},\omega)p_g^{*\perp,n}(\boldsymbol{x};\boldsymbol{y}+\boldsymbol{\eta},\omega)R_{m,n}^{\perp}(\boldsymbol{y},\boldsymbol{\eta},\tau)d\tau d\boldsymbol{\eta} d\boldsymbol{y}$$

- Here, $R_{m,n}^{\perp}(\boldsymbol{y},\boldsymbol{\eta},\tau) = \langle \Theta_m(\boldsymbol{y},\tau), \Theta_n(\boldsymbol{y}+\boldsymbol{\eta},\tau+\Delta\tau)\rangle$
- The above equation is a two point cross – correlation term between continuity – continuity, continuity – momentum, continuity – energy, momentum – momentum, momentum – energy, and energy – energy equations.
- This becomes very long after decomposing it into base, anisotropic, and isotropic fluctuating terms. One of the term in energy – energy correlation has 6561 total terms.

Cross – Correlation Source Term for Shock Noise

- We model source terms as, $R^{\perp}_{m,n}(y,\eta,\tau) = \{R^{\perp}_{m,n}\}R$
- From one of the terms in energy – energy cross - correlation, we choose the following term for shock noise.

$$\left\langle \gamma \frac{\partial \underline{u_j}^{(1)}\, \underline{p}^{(2)}}{\partial y_j}, \gamma \frac{\partial \underline{u_m}^{(2)}\, \underline{p}^{(2)}}{\partial y_m} \right\rangle \longrightarrow R^{\perp}_{m,n} = \gamma^2 \frac{\partial \overline{p}}{\partial y_j}\frac{\partial \overline{p}}{\partial y_m}\hat{u}_j\hat{u}_m\hat{R}$$

- And $\hat{R} = \exp[-|\tau|/\tau_s]\exp[-(\xi - \hat{u}_c\tau)^2/l^2]\exp[-(\eta^2+\zeta^2)/l_{\perp}^2]$
- Substituting this in the spectral density, and integrating it with respect to τ and $\boldsymbol{\eta}$, we find

$$S_4^{\perp}(\boldsymbol{x},\omega) = \iiint_{-\infty}^{\infty} \sqrt{\pi}\gamma^2 l_{\perp}^2 l\tau_s \exp[-\omega^2 l_{\perp}^2 \sin^2\theta\,/4c_{\infty}^2]\,\frac{\partial \overline{p}}{\partial y_j}(\boldsymbol{y})\hat{u}_j(\boldsymbol{y})$$

$$\left[\int_{-\infty}^{\infty} \frac{\partial \overline{p}}{\partial y_j}(k,y_2,y_3)\,\frac{\exp[-l^2(k-\omega\cos\theta\,/c_{\infty})^2/4]}{[1+(1-M_c\cos\theta+u_c k/\omega)^2\omega^2\tau_s^2]}\,dk\right]\hat{u}_j(\boldsymbol{y})d\boldsymbol{y}$$

Ribner, H. S. "Two point correlations of jet noise." *Journal of Sound and Vibration* 56.1 (1978): 1-19.

Cross – Correlation Source Term for Shock Noise

- We use RANS results to estimate the mean quantities, length scales and time scales.

$$\tau_s = 2.25\, k/\epsilon$$
$$l = 1.75\, k^{3/2}/\epsilon$$
$$l_\perp = 0.33\, l$$

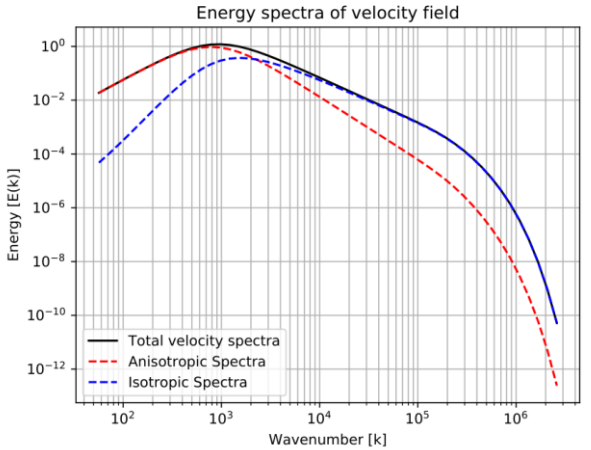
- We estimate the large coherent anisotropic velocities from Pope's composite spectrum.

$$\hat{u} = \int_{\kappa_1}^{\kappa_2} \hat{E}_u\,(y, \kappa, t)\, d\kappa$$

$$\hat{E}_u = c_u\, \epsilon^{1/3} \kappa^{-7/3} f_{L,u}(\kappa, L) f_{\eta,u}(\kappa, \eta)$$

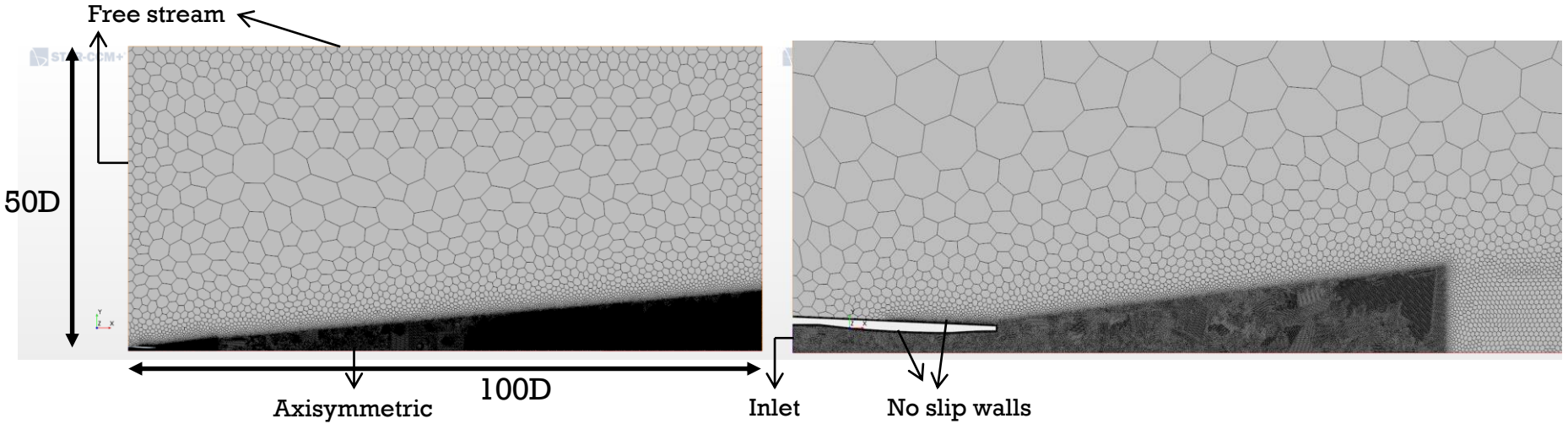
$$f_{L,u}(\kappa, L) = \left(\frac{\kappa L}{\left((\kappa L)^2 + 6.78\right)^{1/2}} \right)^{11/3}$$

$$f_{\eta,u}(\kappa, \eta) = \exp\left[-2.1 \left[\left((\kappa\eta)^4 + 0.0256 \right)^{0.25} - 0.4 \right] \right]$$



Ishihara, Takashi, Kyo Yoshida, and Yukio Kaneda. "Anisotropic velocity correlation spectrum at small scales in a homogeneous turbulent shear flow." *Physical review letters* 88.15 (2002): 154501.

Mesh and Boundary Conditions



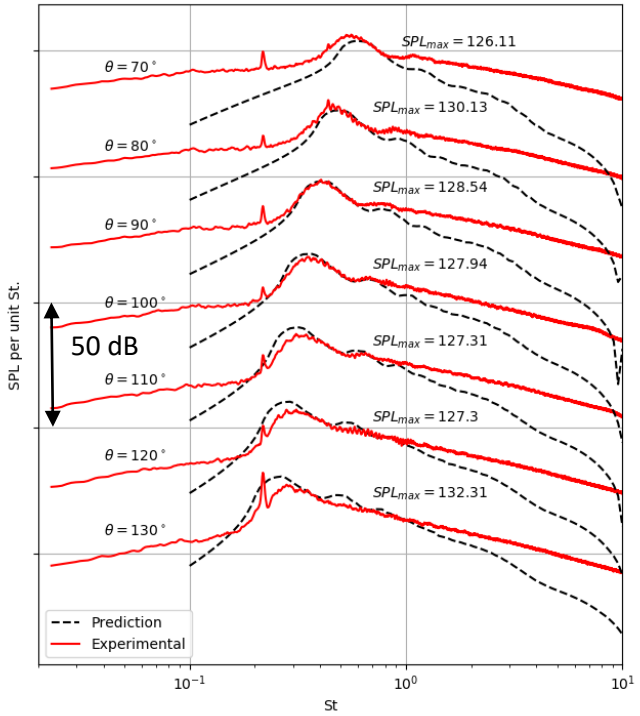
Converging-Diverging Nozzle
Number of vertices : 306851
Number of cells : 156177
Number of faces : 460723

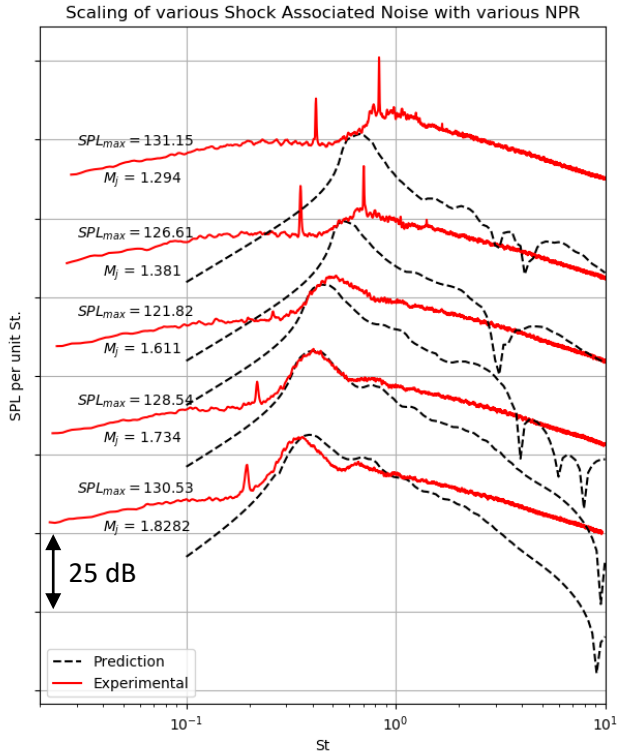
At inlet :
Design Mach number = 1.5
Various nozzle pressure ratios
TTR (total temperature ratio) : 1.0

Outlet : Pressure outlet
Freestream : Mach number = 0.1

Shock Associated Noise Results

$M_d = 1.5\,; M_j = 1.734$





- Observer is at 100D (far-field)
- Experimental data is taken from SHJAR (Small Hot Jet Acoustic Rig) database from NASA Glenn Research Center

Bridges, James, and Clifford Brown. "Validation of the small hot jet acoustic rig for aeroacoustic research." *11th AIAA/CEAS Aeroacoustics Conference*. 2005.

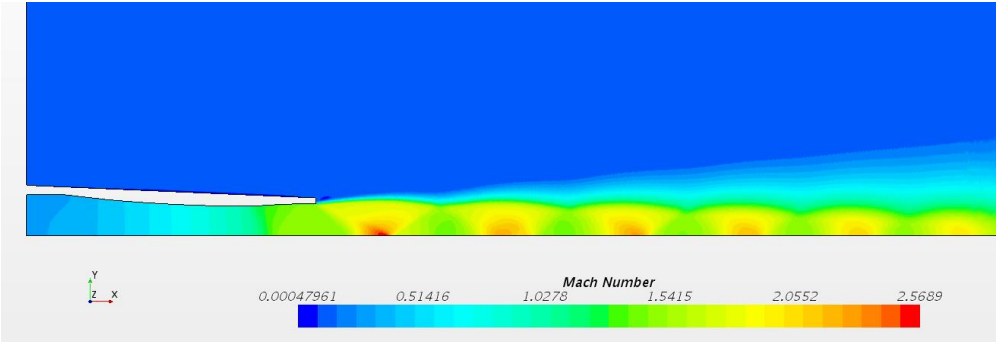
Summary and Future Work

- We used a N-S equations based acoustic analogy to predict the shock noise, and preliminary results are good.
- Using an instability model for estimating the large scale coherent velocities.
- Using LES data to validate which terms from correlation equations contribute the most to shock associated noise.
- Using LES data for prediction by doing numerical two point correlation at shock locations.
- Sensitivity study on how the noise level varies with perturbations of various input parameters.

Thank You!! Questions?

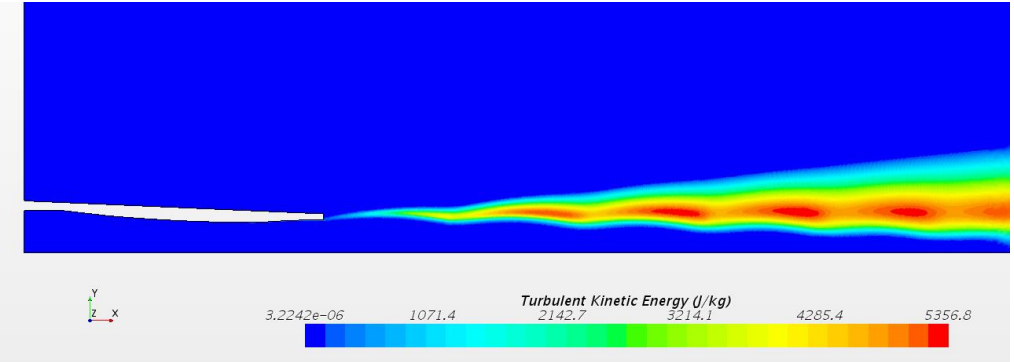
Backup Slides

CFD Results



$M_d = 1.5\ ; M_j = 1.734$

Contour plot of Mach number



Contour plot of TKE

Two Point Cross – Correlation Terms

Continuity – continuity correlation:

$$R^{\perp}_{0,0}(\boldsymbol{y},\boldsymbol{\eta},\tau) = \left\langle \frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial \underline{\rho}^{(2)}}{\partial \tau} \right\rangle + \left\langle \frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial \underline{\rho}^{(2)} \underline{u}_m^{(2)}}{\partial y_m} \right\rangle + \left\langle \frac{\partial \underline{\rho}^{(1)} \underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial \underline{\rho}^{(2)}}{\partial \tau} \right\rangle + \left\langle \frac{\partial \underline{\rho}^{(1)} \underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial \underline{\rho}^{(2)} \underline{u}_m^{(2)}}{\partial y_m} \right\rangle$$

First term of continuity – continuity correlation:

$$\left\langle \frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial \underline{\rho}^{(2)}}{\partial \tau} \right\rangle = \frac{\partial \overline{\rho}^{(1)}}{\partial \tau}\frac{\partial \overline{\rho}^{(2)}}{\partial \tau} + \frac{\partial \overline{\rho}^{(1)}}{\partial \tau}\frac{\breve{\rho}^{(2)}}{\breve{\tau}_s^{(2)}}\breve{R} + \frac{\partial \overline{\rho}^{(1)}}{\partial \tau}\frac{\hat{\rho}^{(2)}}{\hat{\tau}_s^{(2)}}\hat{R} + \frac{\breve{\rho}^{(1)}}{\breve{\tau}_s^{(1)}}\frac{\partial \overline{\rho}^{(2)}}{\partial \tau}\breve{R} + \frac{\breve{\rho}^{(1)}}{\breve{\tau}_s^{(1)}}\frac{\breve{\rho}^{(2)}}{\breve{\tau}_s^{(2)}}\breve{R} + \frac{\breve{\rho}^{(1)}}{\breve{\tau}_s^{(1)}}\frac{\hat{\rho}^{(2)}}{\hat{\tau}_s^{(2)}}\hat{\breve{R}} + \frac{\hat{\rho}^{(1)}}{\hat{\tau}_s^{(1)}}\frac{\partial \overline{\rho}^{(2)}}{\partial \tau}\hat{R} + \frac{\hat{\rho}^{(1)}}{\hat{\tau}_s^{(1)}}\frac{\breve{\rho}^{(2)}}{\breve{\tau}_s^{(2)}}\hat{\breve{R}} + \frac{\hat{\rho}^{(1)}}{\hat{\tau}_s^{(1)}}\frac{\hat{\rho}^{(2)}}{\hat{\tau}_s^{(2)}}\hat{R}$$

$$\left\langle \gamma \frac{\partial \underline{u_j}^{(1)}\, \underline{p^{(1)}}}{\partial y_j}, \gamma \frac{\partial \underline{u_m}^{(2)}\, \underline{p^{(2)}}}{\partial y_m} \right\rangle$$

81 total terms one of which is $\gamma^2 \frac{\partial \overline{p}}{\partial y_j}\frac{\partial \overline{p}}{\partial y_m}\hat{u}_j\hat{u}_m$