Analysis of Shock Associated Noise using Navier-Stokes Acoustic Analogy

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Motivation

- Jet noise has been a major concern in recent years for military and commercial sectors, and having some major ramifications on residential areas.
- Approximately 107,000 sailors are living and working aboard US Navy ships, and 800 million dollars [1] were spent for hearing loss as primary disability of veterans in 2005. In the current decade, that number has risen to over a billion dollars.
- Figure 1 shows a picture of military personnel in close proximity to the aircraft, while figure 2 shows the amount of money spent by US government on hearing disability.

Fig. 1: F/A-18C Hornet on USS Theodore Roosevelt. USN Public release photo: 180123-N-GP724-1102

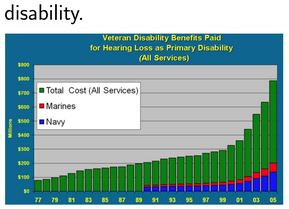


Fig. 2: Amount of money spent on hearing disability [1]

Shock Associated Noise

- Jet noise consists mainly of turbulent mixing noise and shock associated noise.
- Shock noise is generated when the large turbulent structures in the shear layer interact with the shock cells.
- Shock noise is more dominant in the sideline and upstream directions and has a broad peak with different harmonics.
- Figure 3 shows the directivity of various noise sources, while figure 4 shows the noise spectrum in sideline direction.

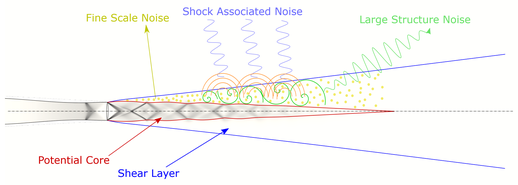


Fig. 3: Directivity of noise sources

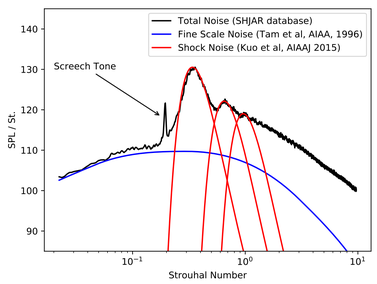
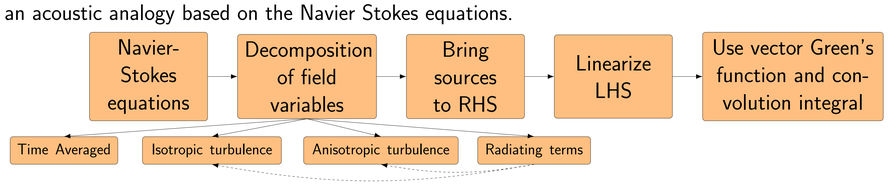


Fig. 4: Empirical noise spectra in the sideline direction ($\theta = 90°$)

Navier-Stokes Acoustic Analogy

Miller [2] proposed an acoustic analogy based on the Navier Stokes equations.



Based on the acoustic analogy, the spectral density of pressure can be written as

$$S_4^{\perp}(\mathbf{x},\omega) = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\sum_{m=0}^{4}\sum_{n=0}^{4}\underbrace{p_g^{*\perp,m}(\mathbf{x};\mathbf{y},\omega)p_g^{\perp,n}(\mathbf{x};\mathbf{y}+\boldsymbol{\eta},\omega)}_{\text{Vector Green's function}}\underbrace{R_{m,n}^{\perp}(\mathbf{y},\boldsymbol{\eta},\tau)}_{\substack{\text{Two point cross correlation term}\\ \langle\Theta(\mathbf{y},\tau),\Theta(\mathbf{y}+\boldsymbol{\eta},\tau+\Delta\tau)\rangle}} d\tau d\boldsymbol{\eta} d\mathbf{y},$$

The source terms from continuity, momentum, and energy equations are given below, where $\underline{q} = \overline{q} + \breve{q} + \hat{q}$

$$\Theta_0 = -\frac{\partial \underline{\rho}}{\partial t} - \frac{\partial \underline{\rho}\underline{u}_j}{\partial x_j} = 0, \qquad \Theta_i = -\frac{\partial \underline{\rho}\underline{u}_i}{\partial t} - \frac{\partial \underline{\rho}\underline{u}_i\underline{u}_j}{\partial x_j} - \frac{\partial \underline{p}}{\partial x_j}\delta_{ij} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \underline{u}_i}{\partial x_j} + \frac{\partial \underline{u}_j}{\partial x_i}\right)\right] - \frac{2}{3}\frac{\partial}{\partial x_j}\left[\mu\frac{\partial \underline{u}_k}{\partial x_k}\right],$$

$$\Theta_4 = -\frac{\partial \underline{p}}{\partial t} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\underline{u}_k\underline{u}_k}{\partial t} - \gamma\frac{\partial \underline{u}_j\underline{p}}{\partial x_j} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\underline{u}_j\underline{u}_k\underline{u}_k}{\partial x_j} + (\gamma-1)\frac{\partial}{\partial x_j}\left[\frac{c_p\mu}{Pr}\frac{\partial \underline{T}}{\partial x_j}\right] + (\gamma-1)\frac{\partial}{\partial x_j}\left[\mu\underline{u}_i\left(\frac{\partial \underline{u}_i}{\partial x_j} + \frac{\partial \underline{u}_j}{\partial x_i}\right)\right] - \frac{2}{3}\delta_{ij}\frac{\partial}{\partial x_j}\left[\mu\underline{u}_i\frac{\partial \underline{u}_k}{\partial x_k}\right],$$

Overall, there are total of 137 correlation source terms of $\underline{q}$. Further expansion to mean, isotropic, and anisotropic quantities result in a lot of terms. One term in energy-energy correlation contains 6561 terms!

Shock Noise Term

One of the terms in energy-energy correlation is the multiplication of anisotropic velocity with the gradient of pressure.

$$\left\langle \gamma\frac{\partial \underline{u}_j^{(1)}\underline{p}^{(1)}}{\partial y_j}, \gamma\frac{\partial \underline{u}_m^{(2)}\underline{p}^{(2)}}{\partial y_m}\right\rangle \rightarrow R_{m,n}^{\perp} = \gamma^2\frac{\partial \overline{p}}{\partial y_j}\frac{\partial \overline{p}}{\partial y_m}\hat{u}_j\hat{u}_m\hat{R}$$

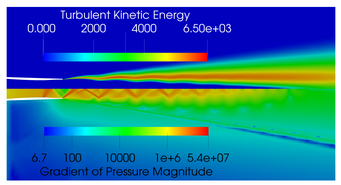


Fig. 5: (Above) Turbulent Kinetic Energy, (Below) Gradient of mean pressure

Figure 6 shows the scaling of the intensity of shock noise term as β^4 which was observed by Haper-Bourne and Fisher [3].

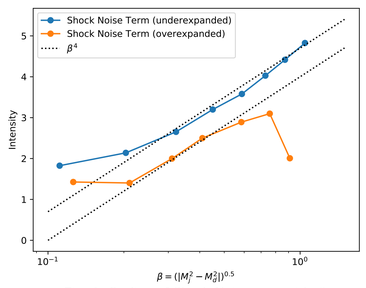


Fig. 6: Scaling of Shock Noise term with β

Prediction Results

Using an appropriate form of normalized two point correlation term, we can analytically integrate the spectral density to get

$$S_4^{\perp}(\mathbf{x},\omega) = \iiint_{-\infty}^{\infty}\sqrt{\pi}\gamma^2 l_{\perp}^2 l\tau_s \exp\left[-\frac{\omega^2 l_{\perp}^2\sin^2\theta}{4c_{\infty}^2}\right]\frac{\partial \overline{p}}{\partial y_j}(\mathbf{y})\hat{u}_j(\mathbf{y})$$
$$\left[\int_{-\infty}^{\infty}\frac{\exp\left[-l^2\left(k - \frac{\omega\cos\theta}{c_{\infty}}\right)^2/4\right]}{\left[1 + \left(1 - M_c\cos\theta + \frac{u_c k}{\omega}\right)^2\omega^2\tau_s^2\right]}\frac{\partial \overline{p}}{\partial y_j}(k, y_2, y_3)dk\right]\cdot\hat{u}_j(\mathbf{y})d\mathbf{y}.$$

We use theory and CFD simulation to evaluate the above equation, and compare the prediction with the experimental results.

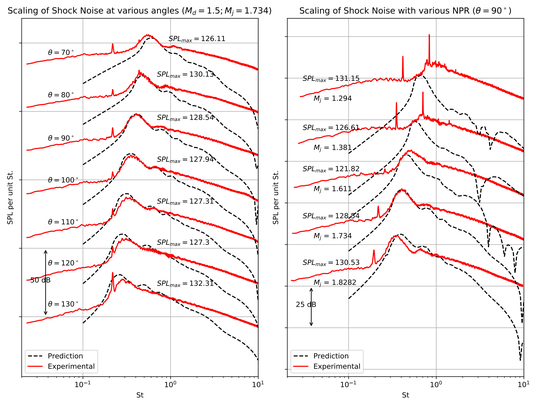


Fig. 7: Broadband shock associated noise prediction using RANS results

Source Location

- The source locations correspond very well when compared to the experiments of Podboy [4].
- There is some discrepancy considering the fact that the RANS results do not capture all the shocks, as well as the refraction effects are not taken into account in the model.

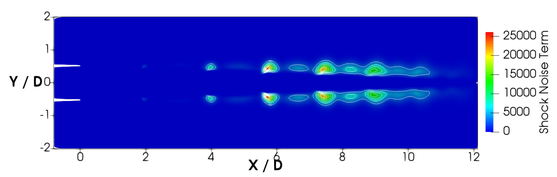


Fig. 8: Source locations for shock noise for NPR=5.2

Conclusions and Future Work

- The shock noise source term contains gradient of pressure which represents the shock strength while the anisotropic velocity represents the large eddies.
- The source term scales very well with off-design parameter, β, as well as the preliminary prediction results from the model are good.
- Perturbing the input parameters to see how it affects the shock noise source term and finding ways to reduce it.
- Using LES data for prediction as well as understanding how the statistics of turbulence change before and after shocks.

Literature Cited

[1] *Acquisition Safety - Noise Control Aboard Navy Ships*. 2014. url: `https://www.public.navy.mil/navsafecen/Pages/acquisition/noise_control.aspx` (visited on 03/21/2019).

[2] S. A. E. Miller. "Noise from Isotropic Turbulence". In: *AIAA Journal* 55.3 (2017), pp. 755–773. issn: 0001-1452. doi: 10.2514/1.J055114.

[3] M. Harper-Bourne and M. J. Fisher. "The Noise from Shocks Waves in Supersonic Jets". In: *NATO AGARD-Cp-131* September 1973 (1974), pp. 11–13.

[4] G. Podboy et al. "Noise Source Location and Flow Field Measurements on Supersonic Jets and Implications Regarding Broadband Shock Noise". In: *NASA-TM-2017-219544* August 2017 (2018).

Acknowledgements

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